

Step Up Your Statistical Practice with Today's SAS/STAT® Software

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Abstract

Has the rapid pace of SAS/STAT® releases left you unaware of powerful enhancements that could make a difference in your work? Are you still using PROC REG rather than PROC GLMSELECT to build regression models? Do you understand how the GENMOD procedure compares with the newer GEE and HPGENSELECT procedures? When should you turn to PROC ICPHREG rather than PROC PHREG for survival modeling?

This paper will increase your awareness of modern tools in SAS/STAT by providing high-level comparisons with well-established tools and explaining the benefits of enhancements and new procedures. The paper focuses on new tools in the areas of regression model building, generalized linear models, survival analysis, and mixed models. When you see the advantages of these tools, you will want to put them into practice. The paper also points out resources that will guide you to new tools in other important areas, such as Bayesian analysis, causal inference, item response theory, methods for missing data, and survey data analysis.

Introduction

Are you a creature of habit when it comes to analyzing data? Do you still rely on PROC REG and PROC GLM for your regression studies because those are the procedures you learned about in school? Have you heard about recent releases of SAS/STAT and new procedures—but not found time to check them out? If so, the procedures that you know best might not be your best choices when compared with newer procedures that deliver significant advances in methodology. And you might not be aware of alternatives that could make a difference in your work.

This paper provides that awareness. Each of its four main sections focuses on an area where SAS/STAT has grown significantly in recent years:

- The section on “[Regression Model Building](#)” describes new tools for selecting the effects in your model when you have many variables to choose from—continuous or categorical. You might be building traditional explanatory models if your goal is to gain insights. Or you might now be building predictive models if your goal is accurate prediction with new data. Either way, you can build better models by applying modern selection methods, such as the lasso, and you can build a broader range of models in which the response can be categorical or continuous.
- The section on “[Inferential Analysis of Generalized Linear Models](#)” describes new tools for different kinds of inference—such as estimation of treatment effects—within the framework of generalized linear models. These tools help you to take advantage of modern Bayesian methods, deal with overdispersion, and handle missingness that is due to dropouts in longitudinal studies.
- The section on “[Survival Analysis](#)” describes new tools for estimation and hypothesis testing and for modeling the outcome of interest when you have time-to-event data. These tools are indispensable for valid inference because they are specialized for particular problems that you encounter with right-censored data, interval-censored data, competing risks, and clustered data.
- The section on “[Analysis of Mixed Models](#)” describes the various procedures available in SAS/STAT software for handling models with both fixed and random effects. Understanding how these tools compare in terms of flexibility and practical advantages will help you decide which ones to apply in your work.

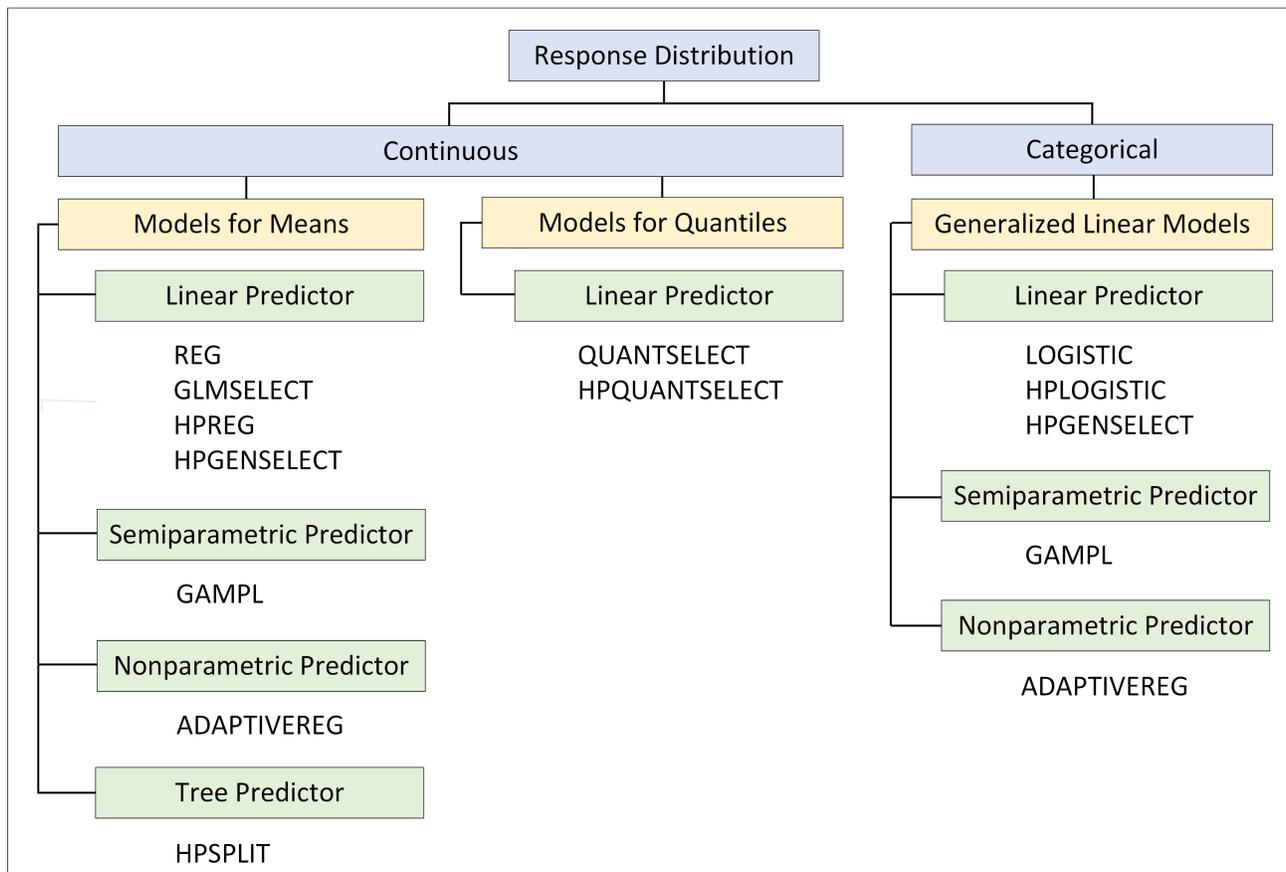
Each section begins by noting the most familiar procedures in that area, and it then presents new tools—enhancements and new procedures—that give you greater flexibility for statistical modeling, specialized inference for complex data, and improved performance for large data. The discussion compares the objectives, assumptions, and benefits of the new tools.

Because this paper presents a high-level view, it does not include examples or explanations of methods. Instead, each section refers to introductory papers that cover these aspects. The final section points out resources that guide you to new tools in areas of SAS/STAT software that are not covered here, such as Bayesian analysis, causal inference, item response theory, methods for missing data, and survey data analysis.

Regression Model Building

One of the most frequently asked questions in statistical practice is the following: “I have hundreds of variables—even thousands, so which ones should I include in my regression model?” [Figure 1](#) presents the various regression model building procedures now available in SAS/STAT, and it shows how they are related to each other.

Figure 1 Procedures for Regression Model Building



The models you can build with these procedures fall into three broad categories: regression models for means of continuous responses, quantile regression models for continuous responses, and generalized linear models for categorical responses. In [Figure 1](#), the term “generalized linear models” refers to regression models for categorical responses that assume a parametric distribution in the exponential family and a corresponding link function.

With the exception of PROC REG, all the procedures in [Figure 1](#) enable you to specify predictors that are continuous or categorical. Most of these procedures build models in which the predictor effects enter the model linearly. The HPSPLIT procedure fits tree models in which the predictor variables enter the model through indicator functions of regions of the predictor space that are defined by variable splits. The GAMPL procedure allows spline functions of continuous predictors in addition to linear predictors, and the ADAPTIVEREG procedure builds predictors that are based entirely on splines. The next sections describe the capabilities and benefits of the model building procedures in each of the three categories.

Building Regression Models for Means of Continuous Responses

The REG procedure has always served the dual purposes of fitting and building standard regressions models, which apply to continuous responses and assume no parametric distribution for the response. However, this procedure is limited to regression models in which the predictors are continuous variables. Longtime users of PROC REG are often surprised to learn that this limitation is overcome by the GLMSELECT procedure, which is now the flagship SAS/STAT procedure for building standard regression models. Keep in mind that the REG procedure is still the preferred tool for fitting standard regression models when you need inferential methods, influence statistics, and diagnostic plots.

A major advantage of PROC GLMSELECT over PROC REG is that it supports effect selection methods for general linear models of the form

$$y_i = \beta_0 + \beta_1 x_{i1} + \dots + \beta_p x_{ip} + \epsilon_i, \quad i = 1, \dots, n$$

where the response y_i is continuous and the predictors x_{i1}, \dots, x_{ip} represent main effects that consist of continuous or classification variables, and interaction effects of these variables. You specify the model by using MODEL and CLASS statements as in the GLM procedure. By using the EFFECT statement, you can include more types of effects—such as polynomial and spline effects—that are constructed from the variables.

Another advantage of PROC GLMSELECT is that it provides lasso methods, introduced by Tibshirani (1996), in addition to the forward, backward, and stepwise selection methods available in the REG procedure. Lasso methods leave all the effects in the model, but they restrict their parameters by setting some to zero while shrinking others toward zero. Thus, they produce models that are sparser and potentially more interpretable (Hastie, Tibshirani, and Wainwright 2015). Table 1 summarizes the selection methods available in the GLMSELECT procedure.

Table 1 Effect Selection Methods in the GLMSELECT Procedure

Method	Description
Forward selection	Starts with no effects and adds effects
Backward elimination	Starts with all effects and deletes effects
Stepwise selection	Starts with no effects; effects are added and can be deleted
Least angle regression	Starts with no effects and adds effects; at each step, estimated β s are shrunk toward 0
Lasso	Constrains sum of absolute β s; some β s set to 0
Elastic net	Constrains sums of absolute and squared β s; some β s set to 0
Adaptive lasso	Constrains sum of absolute weighted β s; some β s set to 0
Group lasso	Constrains sum of Euclidean norms of β s corresponding to effects; all β s for the same effect are set to 0 or are nonzero

The GLMSELECT procedure also provides extensive capabilities for customizing effect selection. You can specify information criteria or criteria based on significance levels. You can also specify criteria based on validation; this approach avoids overfitting the training data by partitioning the data into subsets for training, validation, and testing.

To address the computational demands of selection from a very large number of effects, the GLMSELECT procedure has added screening approaches that you can combine with selection methods to reduce the number of regressors to a smaller subset on which the selection is then performed.

Cohen (2006) provides an introduction to the GLMSELECT procedure, and Cohen (2009) describes its strengths for building models with large data. Günes (2015) discusses regression methods based on penalization. Gibbs et al. (2013) explain the versatility of the EFFECT statement, which is available in many SAS/STAT modeling procedures.

The HPREG procedure is a high-performance procedure that has many of the same features as the GLMSELECT procedure for fitting and building standard regression models. PROC HPREG is referred to as a high-performance procedure because it runs in either single-machine mode or distributed mode, and it is multi-threaded. Cohen and Rodriguez (2013) describe the design of high-performance statistical modeling procedures and discuss when these procedures provide performance benefits.

The HPSPLIT procedure is a high-performance procedure that builds regression trees, which model continuous responses, and classification trees, which model categorical responses. The predictor variables can be categorical or continuous, and the tree is built by recursively splitting the predictor space into nonoverlapping segments, which define the terminal nodes of the tree. The process begins by growing a large, full tree. To prevent overfitting, the full tree is pruned back to a smaller subtree that balances the goals of fitting the training data and predicting new data. The average response of the training observations in a terminal node serves to predict the response for new observations that fall into that node.

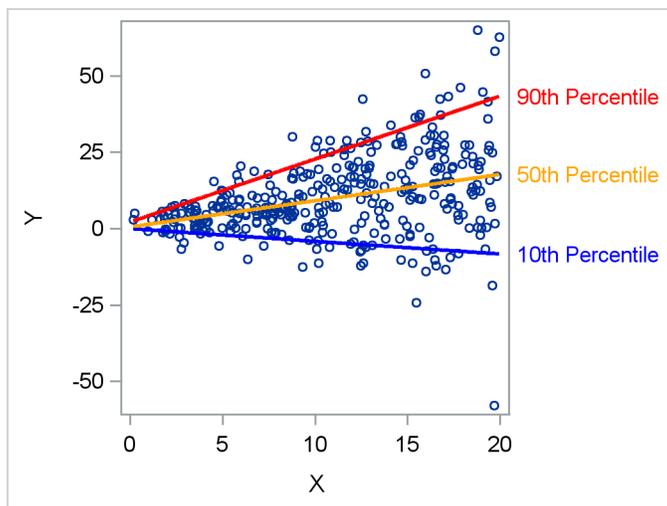
An advantage of regression trees over standard regression models is that they are easy to explain; tree diagrams can be highly interpretable when the tree size is small. On the other hand, because regression trees lack flexibility for capturing smooth relationships between the predictors and the response, they often fail to provide the predictive accuracy of linear regression models.

Building Quantile Regression Models for Continuous Responses

The standard regression models that you build with the GLMSELECT and HPREG procedures predict the conditional mean of the response ($E[Y|X]$), and they assume that the conditional variance of the response is constant ($\text{Var}[Y|X] = \sigma^2$). Models of this type cannot describe data in which the shape of the response distribution depends on the predictors.

For example, consider the data shown in Figure 2, where the variance of Y increases with X . You can use a simple linear regression model to predict $E[Y|X]$, but this model cannot account for the variation in the conditional distribution of Y .

Figure 2 Quantile Regression Models for Three Percentiles



Quantile regression, introduced by Koenker and Bassett (1978), uses a general linear model to fit conditional quantiles—more commonly referred to as percentiles—of the response without assuming a parametric distribution for the response. Figure 2 shows quantile regression lines for the 10th, 50th, and 90th conditional percentiles of Y , fitted with the QUANTREG procedure. By fitting a more extensive set of percentiles, you can describe the entire conditional distribution of Y . When the shape of the conditional distribution varies nonlinearly with the predictors, you can include polynomial or spline effects in the model.

Table 2 summarizes important differences between standard linear regression and quantile regression.

Table 2 Comparison of Linear Regression with Quantile Regression

Linear Regression	Quantile Regression
Predicts the conditional mean $E[Y X]$	Predicts conditional quantiles $Q_\tau[Y X]$
Applies even with small data	Needs sufficient data
Can assume normality	Does not assume a parametric distribution
Sensitive to outliers	Robust to outliers
Computationally inexpensive	Computationally intensive

For many years, quantile regression was impractical because its computational cost was too high when the number of observations was sufficiently large for accurate prediction of quantiles, especially in the tails. Today, however, quantile regression is quite practical—even for very large data—with the algorithms that are available in the QUANTREG and QUANTSELECT procedures. Quantile regression can reveal the effects of predictors on different parts of the response distribution, and it can yield valuable insights in applications such as risk management, where useful information lies in the tails.

The QUANTSELECT procedure performs effect selection for quantile regression. Like the GLMSELECT procedure, it is designed primarily for effect selection, and it offers similar methods of effect selection. The HPQUANTSELECT procedure is a high-performance procedure that provides functionality similar to that of PROC QUANTSELECT for building quantile regression models. See Rodriguez and Yao (2017) for applications of the QUANTREG and QUANTSELECT procedures.

Building Regression Models for Categorical Responses

The LOGISTIC procedure has long been the primary SAS/STAT procedure for analyzing logistic regression models, and it provides some functionality for building models. In contrast, the HPLOGISTIC procedure is a new—and relatively unknown—high-performance procedure that is designed specifically for fitting and building logistic regression models. In particular, PROC HPLOGISTIC provides advantages for building predictive models from large data, as summarized in Table 3.

Table 3 Comparison of Model Building Capability in PROC HPLOGISTIC and PROC LOGISTIC

HPLOGISTIC Procedure	LOGISTIC Procedure
Provides forward, backward, fast backward, and stepwise methods of effect selection	Provides forward, backward, fast backward, and stepwise methods of effect selection
Provides selection criteria based on information criteria, validation, and significance level of score test	Provides selection criteria based on significance level of score test
Partitions the data into subsets for model training, validation, and testing	Uses all the data for model fitting and inference
Creates SAS code for scoring new data	Creates SAS code for scoring new data and provides a variety of postfitting analysis
Runs in single-machine or distributed mode; is multi-threaded	Runs in single-machine mode; is single-threaded

When you have a response variable that is either categorical or continuous and can be described by a generalized linear model, you can fit or build the model by using the HPGENSELECT procedure. Although the GENMOD procedure is well known as a tool for fitting generalized linear models, it does not provide model selection (note that PROC GENMOD has been enhanced with specialized methods for inferential analysis, as discussed on page 7). Table 4 summarizes the differences between the two procedures.

Table 4 Comparison of PROC HPGENSELECT and PROC GENMOD

HPGENSELECT Procedure	GENMOD Procedure
Fits and builds generalized linear models	Fits generalized linear models
Analyzes large to massive data	Analyzes moderate to large data
Designed for predictive modeling	Designed for inferential analysis
Partitions the data into subsets for model training, validation, and testing	Uses all the data for model fitting and inference
Creates code for scoring new data	Creates code for scoring new data and offers a variety of postfitting analyses
Runs in single-machine or distributed mode; is multi-threaded	Runs in single-machine mode; is single-threaded

The HPGENSELECT procedure provides models for standard response distributions in the exponential family, including the binary, binomial, gamma, inverse Gaussian, normal, Poisson, and Tweedie distributions. In addition, the procedure provides multinomial models for ordinal and nominal responses, and it fits zero-inflated Poisson and negative binomial models for count data. For effect selection, the HPGENSELECT procedure provides backward elimination, forward selection, stepwise regression, and the group lasso method. See Johnston and Rodriguez (2015) for an introduction to the HPGENSELECT procedure.

Generalized additive models are extensions of generalized linear models in which the predictors are semiparametric. This means that, in addition to linear predictors, you can specify additive spline terms that characterize nonlinear dependency structures which are either unknown or too complex to be described by parametric terms.

The GAMPL procedure is a new high-performance procedure that fits generalized additive models by using low-rank regression splines (Wood 2003, 2006). PROC GAMPL does not provide effect selection, but it does produce plots that you can use to explore the additive effects of the spline components. These plots can suggest parametric effects—such as quadratic polynomials—for models that you can then build with the HPGENSELECT procedure.

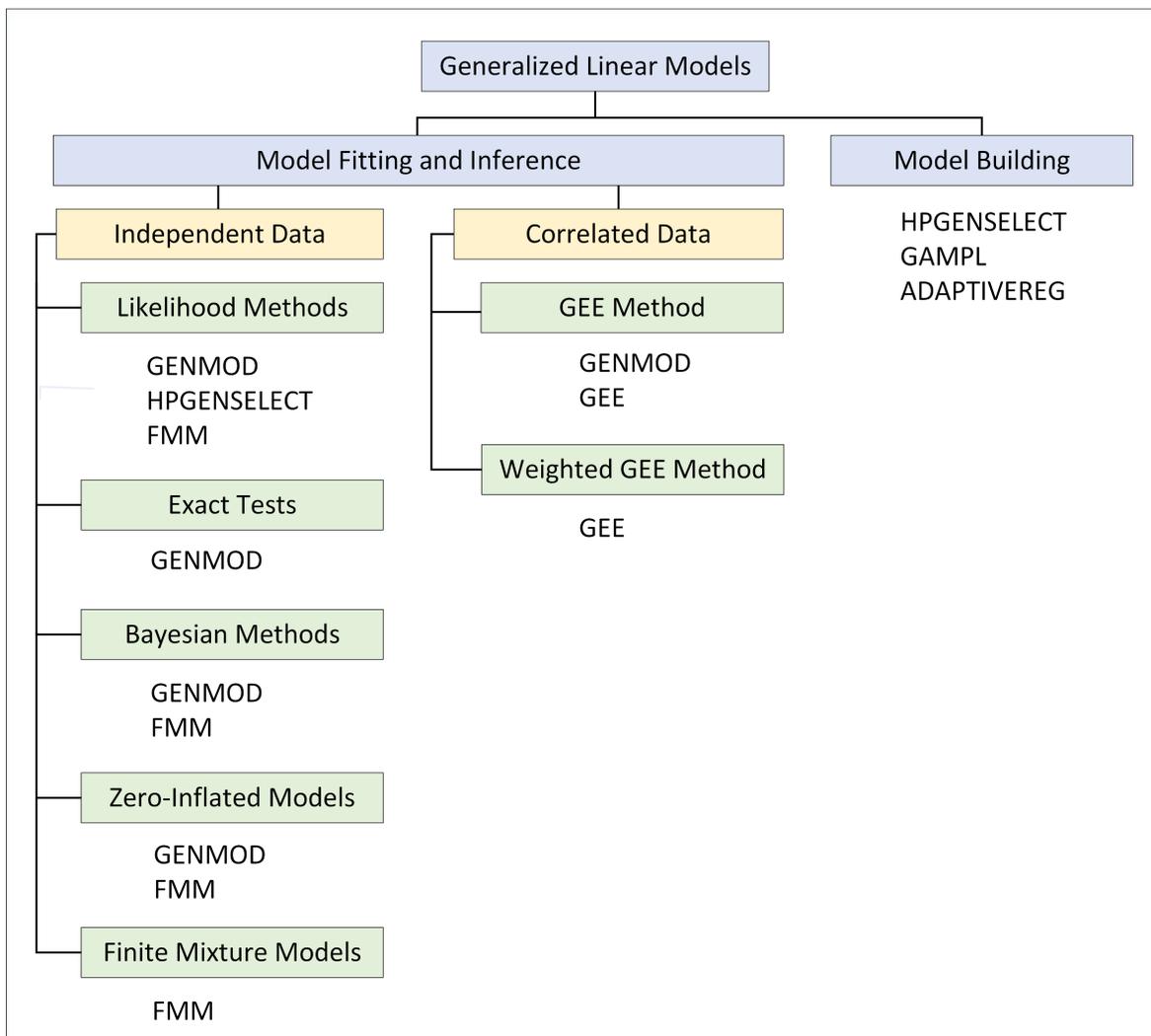
You might be familiar with the earlier GAM procedure for fitting generalized additive models. PROC GAMPL implements newer approaches, such as penalized likelihood estimation, a modified performance iteration method (Wood 2004) and the outer iteration method (Wood 2006). As a result, it provides greatly improved performance for large data.

The ADAPTIVEREG procedure fits response variables with distributions in the exponential family, including the binomial, gamma, inverse Gaussian, normal, negative binomial, and Poisson distributions. The predictor is nonparametric and is constructed from regression splines. The procedure is based on an approach due to Friedman (1991), which constructs spline basis functions in an adaptive way by automatically selecting appropriate knot values for different variables. The approach creates an overfitted model and then prunes it with backward selection. You can use the ADAPTIVEREG procedure to model complex, unknown relationships between the predictors and the response. See Kuhfeld and Cai (2013) for an introduction.

Inferential Analysis of Generalized Linear Models

This section describes procedure enhancements and new procedures for model fitting and inference within the framework of generalized linear models. New procedures for building generalized linear models are described in the preceding section. Figure 3 presents a high-level view of all these procedures.

Figure 3 Procedures for Generalized Linear Models



Generalized linear models assume a parametric response distribution that is in the exponential family. The linear predictor is defined in the same way as for general linear models, and a specified link function expresses how the expected value of the response relates to the linear predictor. [Table 5](#) describes these components.

Table 5 Components of Generalized Linear Models

Component	Description
Linear predictor	Effects involving continuous or classification variables
Link function	Log, logit, inverse, inverse square, and so on
Response distribution	Binary, binomial, gamma, inverse Gaussian, negative binomial, normal, Poisson, Tweedie

A number of widely used statistical models are generalized linear models, including standard linear models with normal errors, logistic regression models for binary data, and log-linear models for multinomial data. You can formulate many other models as generalized linear models by selecting an appropriate response distribution and link function.

The GENMOD procedure is by far the most familiar SAS/STAT procedure for fitting and analyzing generalized linear models. In addition to the models represented in [Table 5](#), PROC GENMOD fits the following extensions:

- models with multinomial response distributions
- models with zero-inflated negative binomial and zero-inflated Poisson response distributions
- models for correlated responses, which the procedure fits by the generalized estimating equation (GEE) method (Liang and Zeger 1986)

Enhancements of the GENMOD Procedure

The GENMOD procedure now provides additional methods of fitting generalized linear models:

- You can request a Bayesian analysis by using the BAYES statement. The model parameters are then treated as random variables, and inferences are based on the posterior distributions of the parameters. The BAYES statement provides a convenient syntax for specifying built-in prior distributions and for requesting credible intervals and summaries of the posterior samples.
Bayesian analysis does not rely on asymptotic approximations, as do likelihood methods. Another benefit of Bayesian analysis is that the results have intuitive interpretations. On the other hand, you must think carefully about your selection of priors, because these can heavily influence the posterior distributions and there is no single correct way to select a prior. Furthermore, you must assess whether the Markov chain that generated the posterior distribution reached stationarity. The BAYES statement produces convergence diagnostics for making this assessment.
- You can request exact conditional Poisson regression, as well as exact binary logistic regression, by using the EXACT statement. Exact conditional inference is based on generating the conditional distribution for the sufficient statistics for the parameters of interest (Cox 1970). This approach is useful in situations involving small samples or small cell counts, where asymptotic properties of maximum likelihood estimation do not apply. The EXACT statement provides exact tests of the parameters for specified effects.
- You can request zero-inflated Poisson regression models or zero-inflated negative binomial regression models with the ZEROMODEL statement. These models are useful when you encounter overdispersion in count data, assuming it results from a process that produces more zero counts than can be explained by the corresponding standard model. An overdispersion diagnostic plot is available for zero-inflated models; it plots the predicted variance as a function of the predicted mean for a zero-inflated response.

Finite Mixture Models

The FMM procedure fits mixtures of generalized linear models by both maximum likelihood and Bayesian techniques, and it models the effects of covariates on both the component distributions and the mixing probabilities. Finite mixture models enable you to account for heterogeneity and overdispersion in your data with a flexible representation that describes the data distribution as a mixture of known distributions.

The FMM procedure provides CLASS and MODEL statements that are familiar from other procedures such as the GLM and GENMOD procedures, and it provides a BAYES statement for requesting built-in Bayesian analysis. The FMM procedure offers a broad selection of distribution functions and automated model selection methods. Kessler and McDowell (2012) provide an introduction to the FMM procedure.

Weighted Methods of Analyzing Missing Data in Longitudinal Studies

Studies of longitudinal data are prevalent in fields such as public health, medical research, and social science. Multiple measurements are taken on the same subject over time in order to discover changes in the response over time and the relationship of changes to covariates (Fitzmaurice, Laird, and Ware 2011). Marginal models are used when population-averaged effects are of interest, and the regression parameters are commonly estimated by the GEE method, which is implemented in the GENMOD procedure.

Missing observations caused by dropouts are a particular concern in longitudinal studies. When the analysis is restricted to complete cases and missingness of responses depends on previous responses, the standard GEE approach can produce biased parameter estimates. The GEE procedure implements inverse probability-weighted GEE methods that account for dropouts under the assumption that data are missing at random (MAR); see Robins and Rotnitzky (1995) and Preisser, Lohman, and Rathouz (2002). These methods can produce unbiased estimates.

Table 6 summarizes the differences between the standard and weighted GEE methods.

Table 6 Comparison of GEE Methods

	Standard GEE Method	Weighted GEE Method
Procedure	GENMOD and GEE	GEE
Data analyzed	Available cases	Available cases
Model specification	Response model Correlation	Response model Correlation Missingness model
Inference assuming data missing completely at random (MCAR)	Valid even when the correlation is misspecified	Valid even when the correlation is misspecified
Inference assuming data missing at random (MAR)	Not generally valid	Valid even when the correlation is misspecified

In addition to GEE methods, the GEE procedure supports the alternating logistic regressions (ALR) algorithm, which is available in the GENMOD procedure and models the association between pairs of responses by using log odds ratios instead of correlations (Carey, Zeger, and Diggle 1993). The GEE procedure provides three methods that are not available in the GENMOD procedure:

- weighted GEE methods
- the ALR method for ordinal multinomial data
- the generalized logit model for nominal multinomial data. Only the independent working correlation structure is supported.

See Lin and Rodriguez (2014) for an introduction to the GEE procedure.

Survival Analysis

Survival analysis deals with time-to-event data that are incomplete due to censoring or competing risks:

- Observations are right-censored when the only information at a given time is that the event of interest has not yet occurred. Likewise, observations are left-censored when the only information at a given time is that the event has already occurred. Observations are interval-censored when the only information is that the event has occurred within a known interval.
- Competing risks are events that impede the observation of the event of interest or that modify the probability that this event will occur. For example, in cardiovascular studies, deaths from other causes such as cancer are considered competing risks.

SAS/STAT software provides specialized procedures for performing survival analysis for right-censored data. Three of these, the LIFETEST, LIFEREG, and PHREG procedures, are particularly well known because they have been available for many years.

The LIFETEST procedure specializes in estimation and hypothesis testing; it computes the Kaplan-Meier estimate of a survivor function and provides the log-rank test for comparing survival curves between groups of observations. The LIFEREG and PHREG procedures specialize in modeling the outcome of interest, but with a clear distinction: PROC LIFEREG fits parametric accelerated failure time (AFT) models, while PROC PHREG fits semiparametric regression models, including the Cox proportional hazards model.

Figure 4 presents a high-level view of all the survival analysis procedures that perform estimation and hypothesis testing.

Figure 4 Survival Analysis Procedures for Estimation and Hypothesis Testing

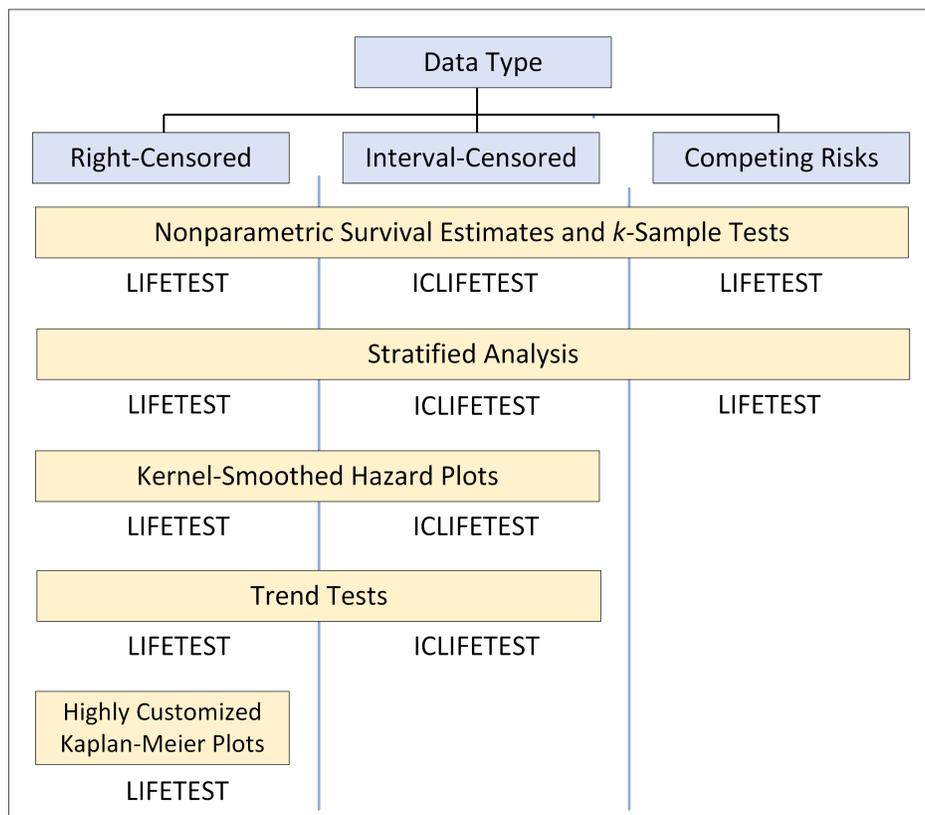
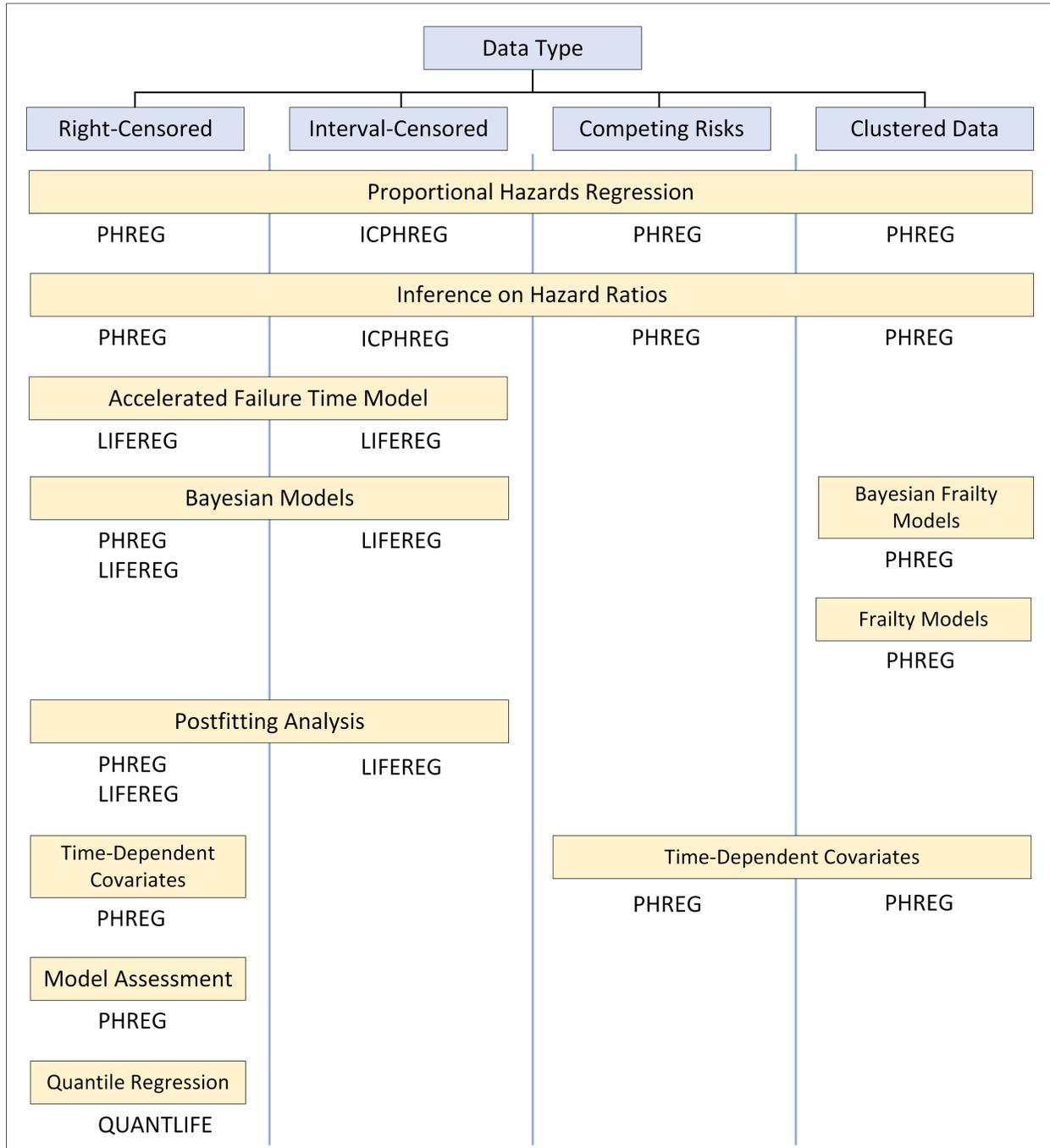


Figure 5 presents a high-level view of all the survival analysis procedures that perform modeling.

Figure 5 Survival Analysis Procedures for Modeling



Enhancements for Survival Analysis

In recent years, the LIFETEST, LIFEREG, and PHREG procedures have been enhanced with specialized methods of analyzing right-censored data:

- The BAYES statement in PROC LIFEREG and PROC PHREG requests a Bayesian analysis of the model; it provides a variety of priors and produces diagnostic plots for convergence assessment.
- The HAZARDRATIO statement in PROC PHREG enables you to request hazard ratios for variables in the model at customized settings, and it provides confidence limits for hazard ratios.

- The PHREG procedure provides methods of model assessment, including the Schemper-Henderson statistic, two versions of concordance statistics, and time-dependent receiver-operator characteristic (ROC) curves.
- The Kaplan-Meier plot that is produced by PROC LIFETEST is now highly customizable through the use of procedure options, graph template modifications, and style template modifications. Kuhfeld and So (2013) provide examples of these approaches.

The survival analysis capabilities of SAS/STAT have also been extended to handle types of time-to-event data other than right-censored data:

- Two new procedures, PROC ICLIFETEST and PROC ICPHREG, specialize in the analysis of interval-censored data and serve as counterparts of PROC LIFETEST and PROC PHREG.
- The LIFETEST and PHREG procedures have been enhanced with specialized methods that analyze the cumulative incidence function (CIF) for competing risks data.
- The RANDOM statement in PHREG provides facilities for fitting frailty models, which handle correlations between failures in clustered data.

Another new procedure, the QUANTLIFE procedure, uses quantile regression to analyze survival data and is particularly useful for modeling heterogeneous data.

Table 7 summarizes the main distinctions among the six procedures that are now available for survival analysis.

Table 7 Comparison of Procedures for Survival Analysis

Procedure	Focus	Inferential Approach	Modeling with Covariates	Censoring Scheme
LIFETEST	Survival function	Nonparametric	No	Right
ICLIFETEST	Survival function	Nonparametric	No	Interval
LIFEREG	Lifetime	Parametric	Yes	Right, left, interval
PHREG	Hazard function	Semiparametric	Yes	Right
ICPHREG	Hazard function	Parametric	Yes	Interval
QUANTLIFE	Lifetime	Semiparametric	Yes	Right

The next three sections explain the benefits of new tools for interval-censored analysis, competing risks analysis, and quantile regression analysis of survival data.

Interval-Censored Analysis

Interval censoring occurs in medical and health studies that involve periodic follow-ups on patients. For example, in acquired immune deficiency syndrome (AIDS) trials, the determination of disease onset is usually based on blood testing, which can only be performed periodically. Interval censoring generalizes left and right censoring. When the left endpoint is zero, the interval represents a left-censored observation. When the right endpoint is infinity, the interval represents a right-censored observation.

Specialized methods of handling interval-censored data have emerged (Turnbull 1976; Finkelstein 1986; Groeneboom and Wellner 1992) and are available in the ICLIFETEST and ICPHREG procedures. These methods offer advantages over midpoint imputation, an ad hoc approach that applies methods for right-censored data to the midpoint of the censoring interval. Chen (2009a) demonstrates that the imputation approach is biased and less efficient than the specialized methods, especially for infrequent or imbalanced assessment.

The ICLIFETEST and ICPHREG procedures resemble the LIFETEST and PHREG procedures in their objectives, but their functionality is relatively limited because fewer methods have been introduced for interval-censored data. The ICLIFETEST procedure provides nonparametric methods of estimating survival functions and statistical testing. The ICPHREG procedure fits proportional hazards regression models and provides inferences based on these models.

For an introduction to the ICLIFETEST procedure, see Guo, So, and Johnston (2014).

Analysis of Competing Risks

Recent enhancements of the LIFETEST and PHREG procedures provide state-of-the-art techniques for the analysis of right-censored data with competing risks. You can use the LIFETEST procedure to perform nonparametric analyses and the PHREG procedure to perform regression analyses.

The concepts of a survival function and a hazard function, which form the basis for standard survival analysis, are inadequate for studying competing risks because once a subject experiences an event other than the event of interest, information about the latter can no longer be ascertained reliably. Instead, the analysis of competing risks is based on the analogous concepts of a cumulative incidence function (CIF) and a cause-specific hazard (CSH) function. The CIF, which is defined as the probability subdistribution function of failure from a specific cause, characterizes the occurrence of a cause-specific outcome over time. The CSH function measures the instantaneous rate of failing from a specific cause in the presence of other causes.

By treating observations of other types of events as censored observations of the event of interest, you can analyze the CSH function for the cause of interest by using certain standard methods, such as the log-rank test in the LIFETEST procedure and Cox regression in the PHREG procedure. However, to analyze the CIF, you need specialized methods, such as those recently provided in the LIFETEST procedure and the PHREG procedure.

The model of Fine and Gray (1999), implemented in the PHREG procedure, extends the Cox model to the CIF setting. The test due to Gray (1988) serves as a counterpart of the log-rank test for testing the equality of CIFs, and is available in the LIFETEST procedure along with a nonparametric estimator of the CIF. You can request CIF analyses in the LIFETEST and PHREG procedures by specifying the code that represents the cause of interest with the EVENTCODE= option. So, Lin, and Johnston (2015) explain how to perform competing risks regression by using the PHREG procedure.

Survival Analysis Based on Quantile Regression

The quantile regression approach to survival analysis, now available in the QUANTLIFE procedure, is useful when you are modeling the survival time and the effects of covariates on the lifetime distribution differ with the covariates. You can use PROC QUANTLIFE to explore such effects—for example, when the variation in the lifetime increases with a continuous covariate.

To decide when to use the QUANTLIFE procedure, you should understand how the quantile regression approach compares with standard methods available in the LIFETEST, LIFEREG, and PHREG procedures. Each method has its advantages and limitations.

Quantile regression is a distribution-free approach in the sense that inference about the regression parameters for a particular quantile of the lifetime depends only on the conditional distribution near that quantile. By comparison, the AFT model in the LIFEREG procedure is more restrictive in its parametric assumption.

Both the Cox proportional model in the PHREG procedure and the AFT model involve an iid error assumption under a suitable transformation of the survival time (Koenker and Geling 2001). This means that covariate effects can shift the location but not the shape of the conditional density for the transformed lifetime. The additional flexibility of quantile regression for modeling the shape can be important when, for example, you are concerned about treatment effects on longer lifetimes.

The QUANTLIFE and LIFEREG procedures both use a regression method to model the lifetime. The LIFEREG procedure provides an efficient estimator for the regression parameters if you are willing to assume a parametric distribution for the lifetime. The regression coefficients computed by the LIFEREG procedure are interpreted as the effect on the mean of the lifetime, and the regression coefficients computed by the QUANTLIFE procedure apply to specified quantiles of the lifetime.

Unlike the QUANTLIFE procedure, the PHREG procedure models the hazard function. Both of these procedures are semiparametric, but in different ways. The Cox model requires no parametric assumption about the baseline hazard function. Another advantage of the Cox model is that it can incorporate time-dependent covariates.

Lin and Rodriguez (2013) provide an introduction to the QUANTLIFE procedure.

Analysis of Mixed Models

When you fit statistical models to data, it is common to assume that all the observations are uncorrelated. The standard linear model procedures—GLM, REG, GLMSELECT, and HPREG—all make this assumption. But when that assumption is violated, it can have a huge impact on the validity of the inferences that you make, and that is when you need mixed models. Mixed models incorporate both fixed effects, which affect only the mean of the response, and random effects, which relate to the covariance between observations.

The MIXED procedure is the flagship SAS/STAT procedure for dealing with linear mixed models. PROC MIXED extends the versatile features for specifying fixed effects in linear models that you find in many SAS/STAT procedures with similarly versatile features for specifying how random effects induce correlation. Likewise, PROC MIXED extends the inferential tools for linear models with fixed effects—for example, Type 3 tests, tests for linear contrasts, and LS-means—with tests and methods that are appropriate for correlation structures.

Although PROC MIXED provides the generality that you need for model estimation and postfit inference, it is not computationally efficient for certain important special cases, including the following:

- linear mixed models with thousands of levels for the fixed and/or random effects
- linear mixed models with hierarchically nested fixed and/or random effects, possibly with hundreds or thousands of levels at each stage of the hierarchy

For these models, which are large and sparse, you need specialized methods. The HPMIXED procedure implements these methods by taking advantage of sparse matrix techniques. PROC HPMIXED does sacrifice certain inferential tools that are available in PROC MIXED but cannot be implemented sparsely. However, if your mixed models fall into these special categories, PROC HPMIXED can often run much faster than PROC MIXED. Wang and Tobias (2009) and Kiernan, Tao, and Gibbs (2016) describe situations that call for the methods in PROC HPMIXED, and they discuss the substantial gains in performance that it provides.

Both the MIXED and HPMIXED procedures deal with responses and random effects that are assumed to be normally distributed. If your response has a nonnormal distribution that belongs to the exponential family—for example, if it has a binary logistic or Poisson distribution—then you need the GLIMMIX procedure, which fits generalized linear mixed models. PROC GLIMMIX can use pseudo-likelihood or marginal maximum likelihood estimation to fit mixed models with a variety of nonnormal error distributions. Schabenberger (2005) outlines the capabilities of PROC GLIMMIX and provides examples that demonstrate its great flexibility for modeling correlated data.

If you need to fit linear mixed models, then one of the three procedures discussed so far in this section—MIXED, HPMIXED, and GLIMMIX—is what you need. But what if you need to fit a random coefficients model in which the coefficients enter the model nonlinearly? Or what if you are fitting a nonlinear mixed model for a pharmacokinetic application where the likelihood depends on solving a system of differential equations? The NLMIXED procedure can fit such models; it enables you to specify a distribution for the response, conditional on the random effects, that has a standard form—such as normal, binary, or Poisson—or a general form that you express with SAS programming statements.

Although both PROC GLIMMIX and PROC NLMIXED enable you to fit models for nonnormal responses, the estimation methods they use require the random effects to be normally distributed. If you need to fit models with nonnormal random effects, then you need to move beyond likelihood methods and consider Bayesian methods, for which the MCMC procedure is the most versatile procedure in SAS/STAT.

Instead of maximizing a likelihood, the Bayesian approach treats all unknown quantities in a model, including both the fixed and random effects, as random variables. The objective is to estimate the joint posterior distribution, often by using the Markov chain Monte Carlo approach (Gelfand et al. 1990). The marginal distribution of the fixed-effects parameters is obtained by using a numerical Monte Carlo method that is based on the Markov chain samples.

In PROC MCMC, you specify the details of a Bayesian model with a combination of procedure statements (such as the PARMs, PRIOR, MODEL, and RANDOM statements) and SAS programming statements. Like the NLMIXED procedure, the MCMC procedure does not assume linearity and it handles a wide range of models. The complex Bayesian models that you can fit with PROC MCMC include linear, generalized linear, and nonlinear random-effects models.

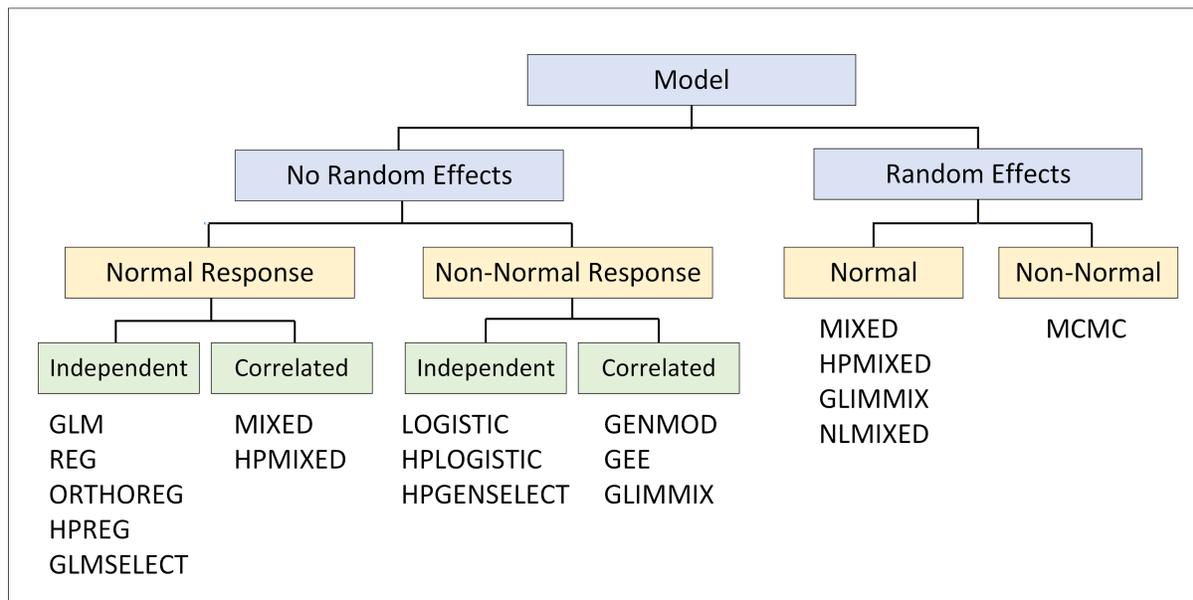
The tutorial papers by Chen (2009b, 2011, 2013) introduce the MCMC procedure. Chen, Brown, and Stokes (2016) offer guidance on using PROC MCMC to perform Bayesian analysis of models for which PROC MIXED and PROC

GLIMMIX implement likelihood methods. Chen and Stokes (2017) illustrate the use of PROC MCMC to fit various multilevel hierarchical models that incorporate complex structures and data dependencies.

In principle, the MCMC procedure is the most general SAS/STAT procedure for analyzing mixed models. It can handle normal data and linear models (like the MIXED procedure), nonnormal data and generalized linear models (like the GLIMMIX, GENMOD, and GEE procedures), and nonlinear models (like the NLMIXED procedure). In addition, the MCMC procedure can handle nonnormal random effects, multilevel random effects, and missing data in ways that are not available with the other procedures. Of course, with this generality you give up much of the convenience and specific analytic features of the more specialized procedures. Thus, the MIXED, HPMIXED, GLIMMIX, and NLMIXED procedures should be your go-to tools for most practical applications of mixed models. The MCMC procedure should be in your toolbox for applications that the other procedures cannot handle, and for situations in which Bayesian modeling is appropriate.

Figure 6 presents a high-level view of procedures for mixed models along with procedures for linear models.

Figure 6 Procedures for Analysis of Linear Models and Mixed Models



Procedures are listed in categories where they provide strengths in practice (in principle, some procedures could be listed in other categories). Procedures within a category have their own unique strengths. For example, the GLM procedure is distinguished from the ORTHOREG, HPREG, and GLMSELECT procedures by its capability for multivariate analysis of variance (MANOVA), which can be regarded as a forerunner to mixed modeling.

Postfitting Analysis of Linear Mixed Models

There is a general-purpose feature for handling linear mixed models that you may not be familiar with—but should be. By using the PLM procedure after you fit the model, you can do additional analysis without rerunning the procedure (either PROC MIXED or PROC GLIMMIX) that you originally used to fit the model and without access to the original data. PROC PLM provides you with two important features:

- Additional analyses, which you can request with the ESTIMATE, LSMEANS, LSMESTIMATE, SLICE, and TEST statements in PROC PLM—even if those statements are not available in the procedure that fit the model.
- Plots of your model results, which you can request with the EFFECTPLOT statement in PROC PLM or the plots that are available through the ESTIMATE, LSMEANS, LSMESTIMATE, and SLICE statements.

In short, PROC PLM enables you to explore the analysis of your model without refitting it—and that makes it a procedure worth learning about. Tobias and Cai (2010) explain how PROC PLM offers these same features not just for linear mixed models, but for a wide spectrum of linear models that you can fit with other procedures, including the GEE, GENMOD, GLM, GLMSELECT, LIFEREG, LOGISTIC, ORTHOREG, and PHREG procedures.

Summary

This paper describes important new tools that SAS/STAT has added in four areas:

- Regression model building
- Inferential analysis of generalized linear models
- Survival analysis
- Analysis of mixed models

If you are over-relying on the basic tools in these areas—perhaps the well-known procedures that you learned about in school—then you will want to explore the enhancements and new procedures discussed in this paper. [Table 8](#) summarizes the many benefits of adopting these tools in your statistical practice.

Table 8 Benefits of Procedure Enhancements and New Procedures in SAS/STAT Software

Benefit	Method	Procedure
Improved predictive ability and interpretability of regression models	Data partitioning	GLMSELECT, HPREG, HPSPLIT, QUANTSELECT, ADAPTIVEREG, HPLOGISTIC, HPGENSELECT
	Lasso methods and information criteria	GLMSELECT, QUANTSELECT, HPGENSELECT
Regression model building for a variety of response types and for complex dependence structures	Categorical responses	HPLOGISTIC, HPGENSELECT, GAMPL, ADAPTIVEREG
	Quantile regression	QUANTSELECT, HPQUANTSELECT
	Regression trees	HPSPLIT
	Spline effects	GLMSELECT, GAMPL, ADAPTIVEREG
Advantages of Bayesian methods, including model versatility, highly interpretable results, and no requirement of a large sample	Generalized linear models	GENMOD
	Survival analysis models	LIFEREG, PHREG, MCMC
	Finite mixture models	FMM
	Mixed models	MCMC
	General Bayesian models	MCMC
Inference for special generalized linear models	Models for overdispersion	GENMOD, FMM
	Exact methods for small samples	GENMOD
	Weighted GEE methods for dropouts in longitudinal data	GEE
Inference for special types of time-to-event data	Methods for interval-censored data	ICLIFETEST, ICPHREG
	Analysis of competing risks	LIFETEST, PHREG
	Analysis of heterogeneous data	QUANTLIFE
High-performance computing for large data	Regression model building	HPREG, HPSPLIT, HPQUANTSELECT, HPLOGISTIC, HPGENSELECT
	Generalized additive models	GAMPL
	Regression trees	HPSPLIT
	Large, sparse mixed models	HPMIXED

Keeping Up with New Releases of SAS/STAT

The new tools discussed in this paper are only a portion of the many enhancements that you will find in recent releases of SAS/STAT software, which are listed in Table 9. The current release is SAS/STAT 14.2.

Table 9 Recent Releases of SAS/STAT Software

Release	Year	Overview	Base SAS Version
SAS/STAT 12.1	2012	Stokes et al. (2012)	SAS 9.3
SAS/STAT 12.3	2013	Stokes (2013)	SAS 9.4
SAS/STAT 13.1	2013	Rodriguez (2014)	SAS 9.4M1
SAS/STAT 13.2	2014	Stokes, Güneş, and Chen (2014)	SAS 9.4M2
SAS/STAT 14.1	2015	Stokes and Statistical R&D Staff (2015)	SAS 9.4M3
SAS/STAT 14.2	2016	support.sas.com/statistics	SAS 9.4M4

The best place to find out about the enhancements in the release that you have is the chapter “What’s New in SAS/STAT” in the online documentation at <http://support.sas.com/statdoc/>. Also, be sure to visit the Statistics and Operations Research focus area at <http://support.sas.com/statistics>. There you can watch helpful videos, download overview papers, and subscribe to a quarterly e-newsletter.

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