Scoring Models, Probability Transformations & Model Calibration Using SAS

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Questions

How can we estimate campaign's Take-up-rate?

How can we transform predictive model propensity scores into probabilities?

What can we do next?

Predictive Modeling Process

What's missing?

Model
Training + Validation (Testing) on a sample

Deploy
As per rank-ordering of **propensity** scores

Validate
Performance of model **propensities**, e.g. lift

Probability Estimates

Training sample may be biased!

Why Probabilities?

- Model scores are propensities of an event when estimated from a biased training sample
  - Propensity scores can only be used for rank-ordering
  - Scores **<u>can not</u>** be used to estimate
    - Expected Sales
    - Expected Profit
    - Optimization of offer allocation does not work

Expected Sales = $\sum_{i=1}^{N} p_i(1) + (1 - p_i)(0)$

= $\sum_{i=1}^{N} p_i$

= NT

Campaign Profit = $\sum_{i=1}^{N} (p_i R - C) = NTR - NC$ [1]

where,

P1 = probability of purchase

N = Total number of customers

T = Overall Take Up Rate (fraction of target customers with desired event)

NT = number of sales

R = Revenue generated from an accepted offer

C = Cost of making an offer

- Bayesian Posterior Probabilities (SAS Decision Node)

Posterior probabilities are adjusted for priors as follows. Let:

t	*be an index for target values (classes)*
oldPrior	*be the old probability or implicit prior probability for target t*
OldPost(i,t)	*be the posterior probability based on OldPrior(t)*
Prior(t)	*be the new prior probability desired for target t*
Post(i,t)	*be the posterior probability based on Prior(t)*

then,

$$Post(i,t) = \frac{OldPost(i,t)Prior(t)\ /\ OldPrior(t)}{\sum_{j} OldPost(i,j)Prior(j)\ /\ OldPrior(j)} \qquad [3]$$

- # Logistic Regression intercept transformation

$$\ln\left(\frac{Post(i,t)}{1-Post(i,l)}\right) = \ln\left[\left(\frac{Post(i,t)}{OldPost(i,l)}\right) x \left(\frac{1-OldPost(i,t)}{1-Post(i,l)}\right)\right] + \ln\left(\frac{OldPost(i,t)}{1-OldPost(i,l)}\right)$$

$$= \gamma + \ln\left(\frac{OldPost(i,t)}{1-OldPost(i,l)}\right)$$

where

Post(i,t) = probability of an event t for i'th customer in the universe

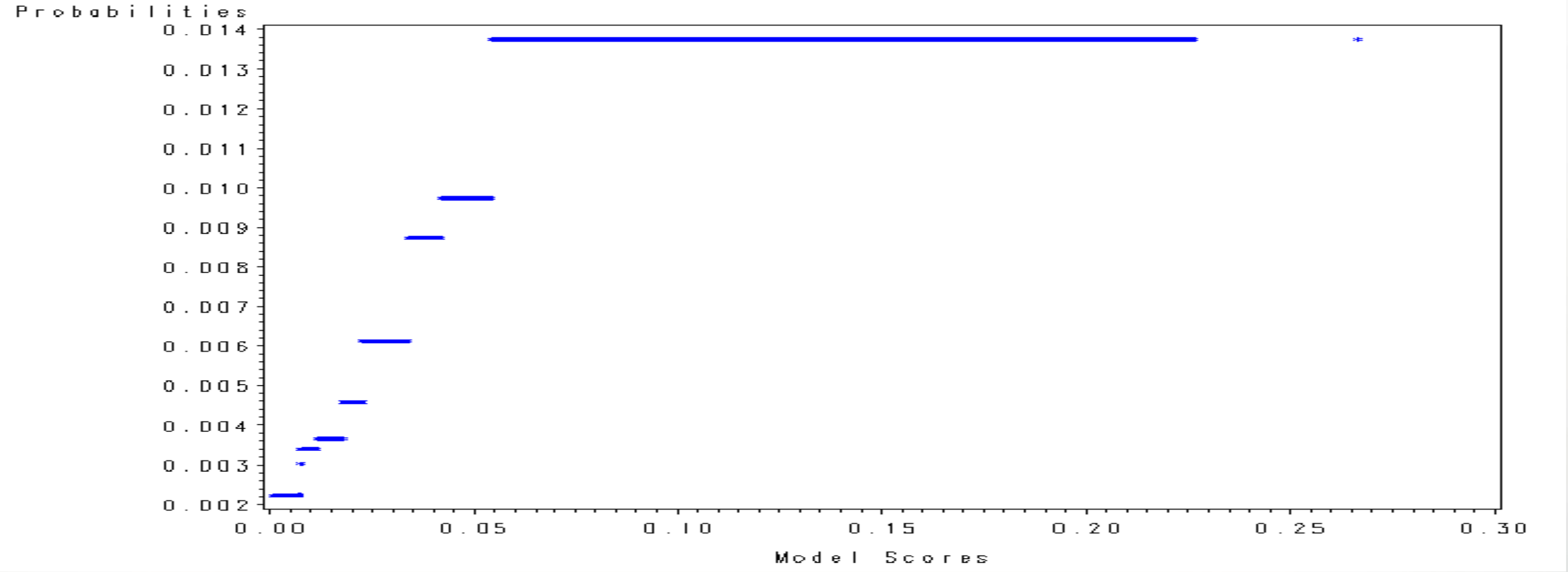
OldPost(i,t) = probability of an event t for i'th customer in the sample

In order to estimate new prediction probabilities only an intercept of the old logistic regression model needs to be adjusted by a constant γ term.

Smooth curves!

Why Probability Smoothing?

Mapping scores' ranks (e.g. deciles) to events empirical Take Up Rates (TUR) in *k* bins, yields a step function:



$$map\ f: x_i \longrightarrow p_j \qquad if\ x_i\ is\ in\ b_j\ bin$$

$$f(x_i) = p_j$$

where,

$$i = 1 \ldots n\ (customers)$$
$$j = 1 \ldots k\ (bins)$$

p_j is a proportion of events in j^{th} bin

Why Smoothing? Heuristic Motivation:

- A step function is:
  - a "crude" approximation of "true" probabilities
  - Discontinued at bin's limits
  - Constant in bins thus not reflecting our expectation of probabilities being monotonic in scores
  - Does not look "smooth"
- One way to improve approximation to "true" probabilities, is to **smooth the step function via Cubic Monotonic Splines**

Probability Smoothing Methodology:

SAS Procedure: **PROC TRANSREG** (*transformation regression*)

Procedure which fits linear models, optionally with spline and other nonlinear transformations of y and x's. [2]

Syntax: TRANSREG Procedure

The following statements are available in PROC TRANSREG:

```
PROC TRANSREG <DATA=SAS-data-set>
<PLOTS=(plot-requests)>
<OUTTEST=SAS-data-set> <a-options> <o-options> ;
      MODEL <transform(dependents </ t-options>)>
      <transform(dependents </ t-options>) ...=>
      transform(independents </ t-options>)
      <transform(independents </ t-options>) ...> </ a-options> ;

      OUTPUT <OUT=SAS-data-set> <o-options> ;

      ID variables ;

      FREQ variable ;

      WEIGHT variable ;

      BY variables ;
```

Probability Smoothing Methodology: Splines

Monotone cubic spline smoothing

A **cubic smoothing spline** consists of cubic polynomials *,
one polynomial between each successive pair of knots,
that have continuous second derivatives at the knots.

This means that each polynomial segment can connect with the next in such a way that their slopes and curvatures change continuously at the knots, resulting in a smooth transition from segment to segment through knots.

Monotonic spline is assumed here to be a non-decreasing spline function.

A *k*th order polynomial (we used k=3) in x is defined as

(*) $$y = \beta_0 + \beta_1 x \quad + \beta_2 x^2 \ldots \beta_k x^k$$

Probability Smoothing Methodology: contn'd

Sample SAS code

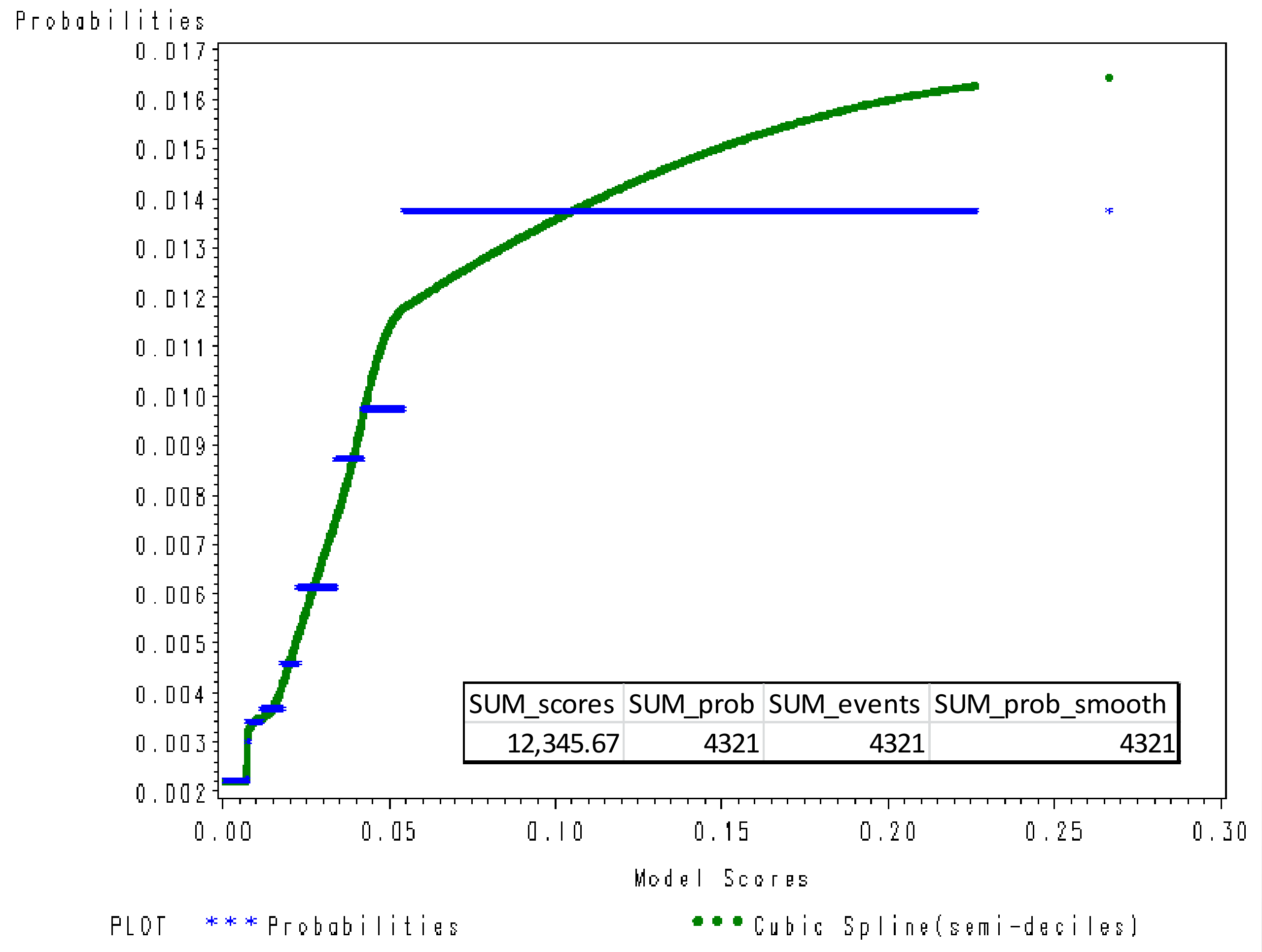
```
PROC TRANSREG DATA=inputDS;
  MODEL IDENTITY(Y) = MSPLINE(X / NKNOTS=9);
  OUTPUT OUT=outputDS PPREFIX=PRED_;
RUN;
```

Where,

- MSPLINES = Monotonic Spline Regressions
- X = model scores
- Y = estimated probabilities

Smoothed Step Function:

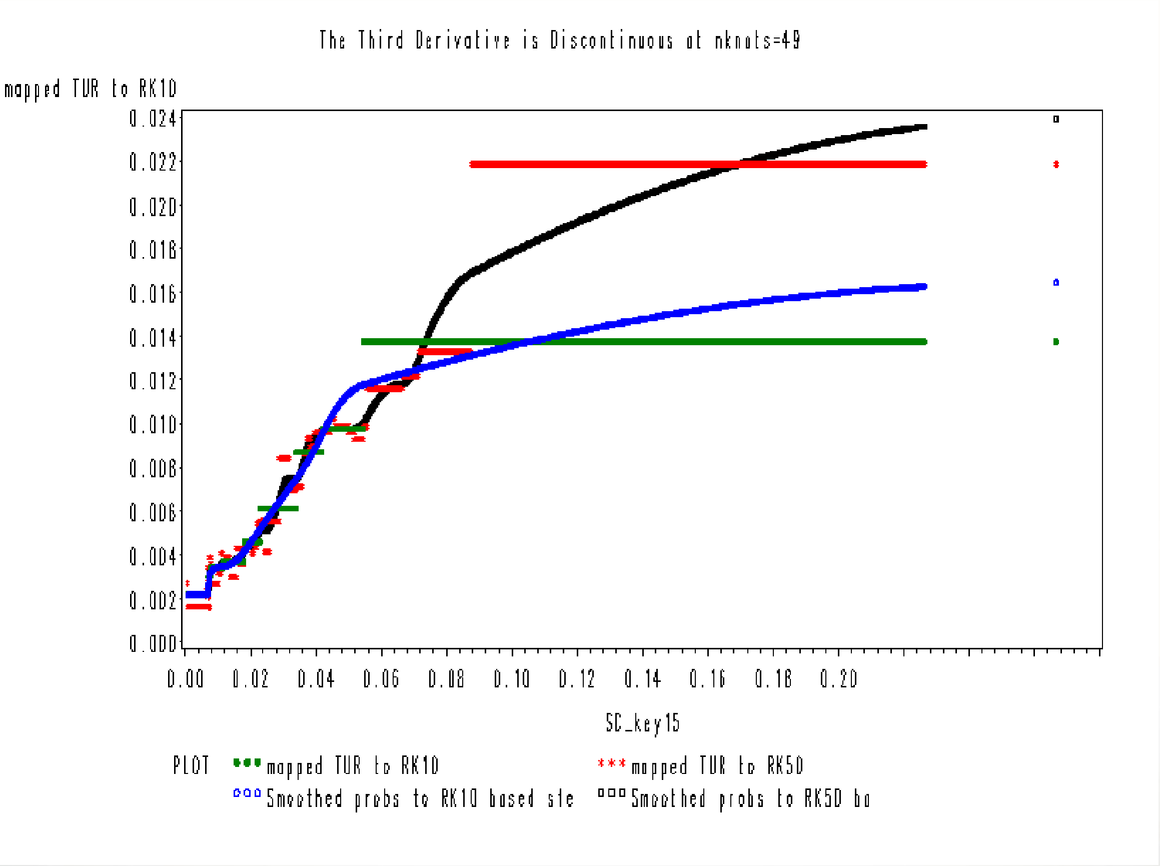
An artificial data example with 9 knots



Probability Calibration

Smoothed Step Function:

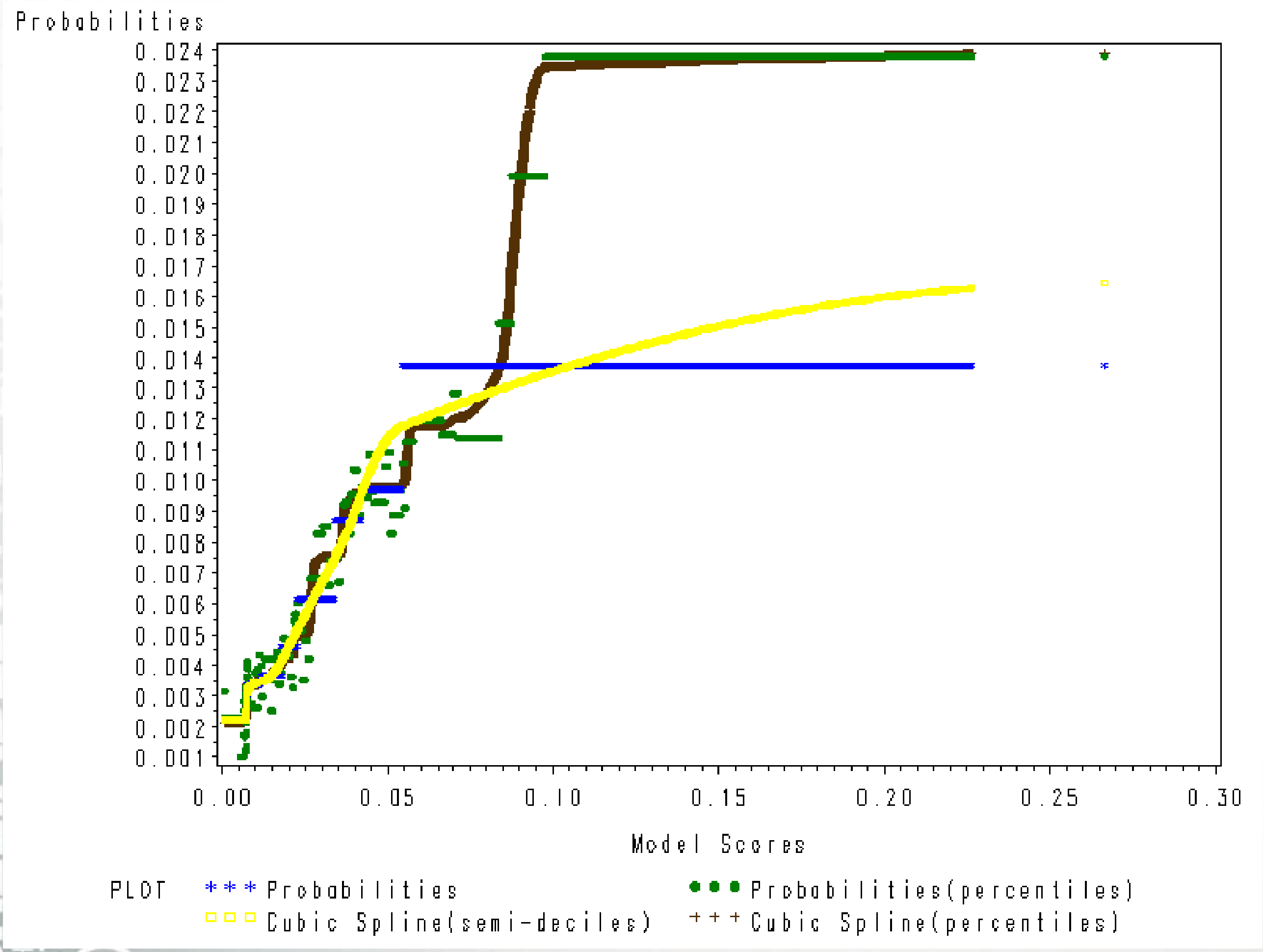
An example with 49 knots



Step function is no longer monotonic. An approximation is closer with more knots but only in the top bins.

Smoothed Step Function:

An example with 99 knots



An approximation with more knots becomes “noisy” in the lower bins but works well in upper bins. More bins better?

Conclusions/next steps

- It appears that empirical mapping and splines are effective in providing non-decreasing properly calibrated probabilities
- Methodology is easy to implement in SAS
- It may be worthwhile to research how to:
  - compare a global transformation to posterior probabilities with monotonic splines
  - Construct optimal binning method

References:

[1] Gregory Piatetsky-Shapiro, Brij Masand , "**Estimating Campaign Benefits and Modeling Lift**"

http://www.kdnuggets.com/gpspubs/kdd99-estimating-campaign-benefits-modeling-lift.pdf

[2] **SAS/STAT(R) 9.2 User's Guide**, Second Edition, Syntax: TRANSREG Procedure

http://support.sas.com/documentation/cdl/en/statug/63033/HTML/default/viewer.htm#statug_transreg_sect005.htm

[3] SAS Enterprise Miner 6.2 – Help documentation