Practice in Growth Curve Modeling

for Women's Menstural Cycle
- Fundamental Overview -

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Introduction

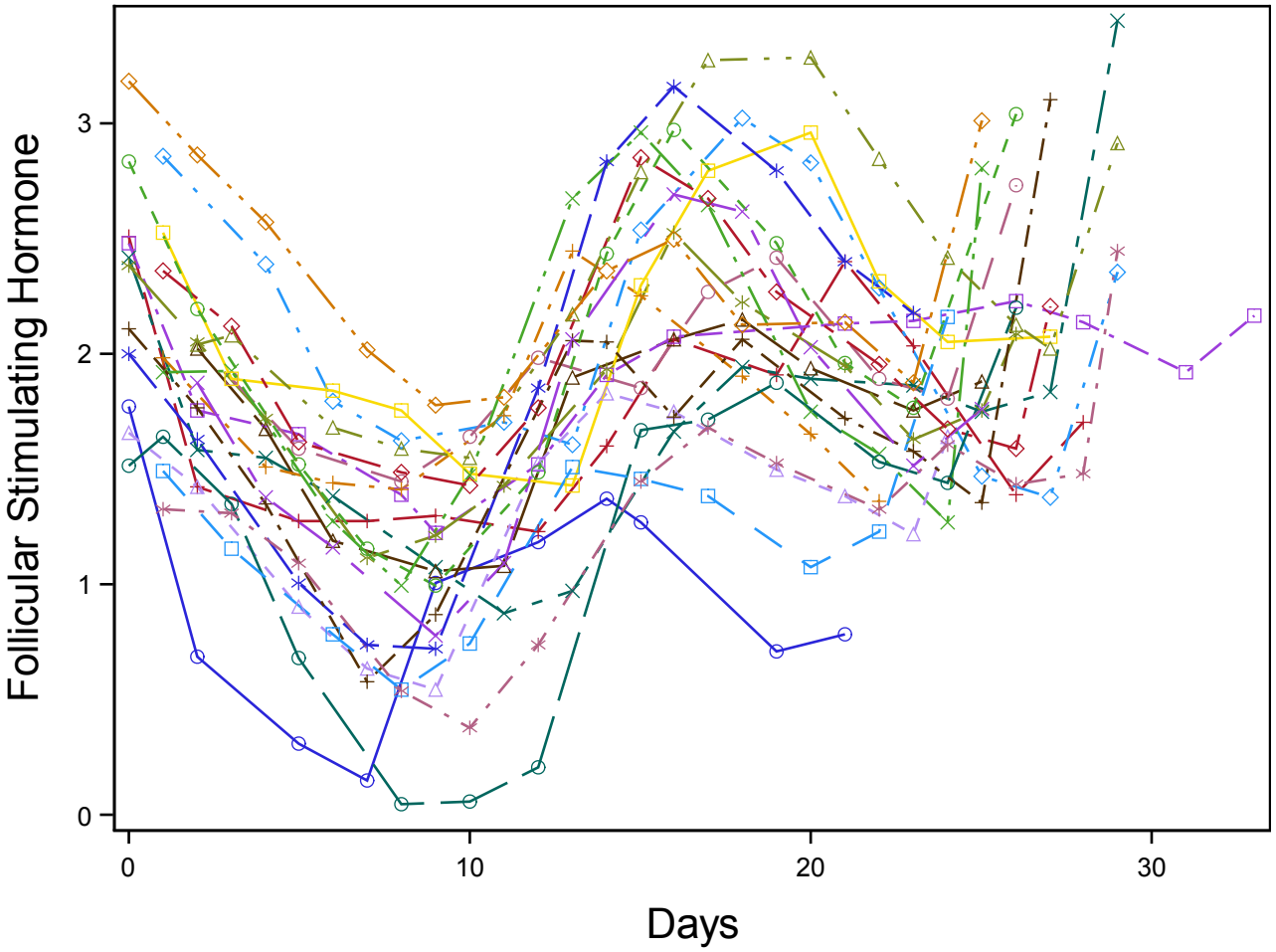
"Essentially, all models are wrong, but some are useful."

(Box and Draper, 1986, p. 424)

- It is a best guess.
- It is an approximation.
- It would be a translation of observation into a model.

Hormones Involved in Women's Menstrual Cycle

Subject-specific Trajectory (in Logarithm Scale)



Linear Mixed-Effect Models (Singer, 1998)

Level 1 (within person)

$$Y_{ij} = \pi_{0j} + \pi_{1j}(TIME)_{ij} + r_{ij}, \quad \text{where } r_{ij} \sim N(0, \sigma^2)$$

Level 2 (between-person)

$$\begin{aligned} \pi_{0j} &= \beta_{00} + u_{0j}, \quad \text{where } \begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix}\right] \\ \pi_{1j} &= \beta_{10} + u_{1j}, \end{aligned}$$

Unconditional Linear Growth Curve

$$Y_{ij} = \underbrace{[\beta_{00} + \beta_{10} TIME_{ij}]}_{\text{FIXED}} + \underbrace{[u_{0j} + u_{1j} TIME_{ij} + r_{ij}]}_{\text{RANDOM}}$$

Note that a 3-level model can be made if individuals within groups are tracked over time.

Using PROC MIXED in SAS

Commands for a linear time trend model

```
PROC MIXED DATA = datafile;
  CLASS id time;
  MODEL y = time / SOLUTION CHISQ;
  REPEATED time / TYPE=UN SUBJECT=id;
RUN;
```

Commands for a subject-specific model

```
PROC MIXED DATA = datafile;
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Concerns Bothering Us

We are working with ...

- A small size of participants with many repeated measurements,

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- An unbalanced data set observed at unequally spaced times, and
- An incomplete data due to missing values.

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We are faced with difficulties of ...

- Determining the level of a model of interest,

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- Applying either or both models,
- Choosing an appropriate pattern of covariance structure, etc.

Parameteric Approaches to Modeling Curves

Nonlinear Model

$$y_i = u(x_i) + \epsilon_i, \quad i = 1, 2, \ldots, n$$

- p-th Degree Polynomial Regession Model (Johnson et al., 2013)

$$u(x_i) = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \cdots + \beta_p x_i^p$$

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- Trigonometric Model (Cornelissen Germaine, 2014)

$$u(x_i) = \mu + A\cos(wx_i + \phi)$$

Modeling Techniques

Piecewise Modeling Approach (Naumova et al., 2001)

$$u(x_i) = \beta_0 + \beta_1 x_i + \beta_2 (x_i - c)\delta,$$

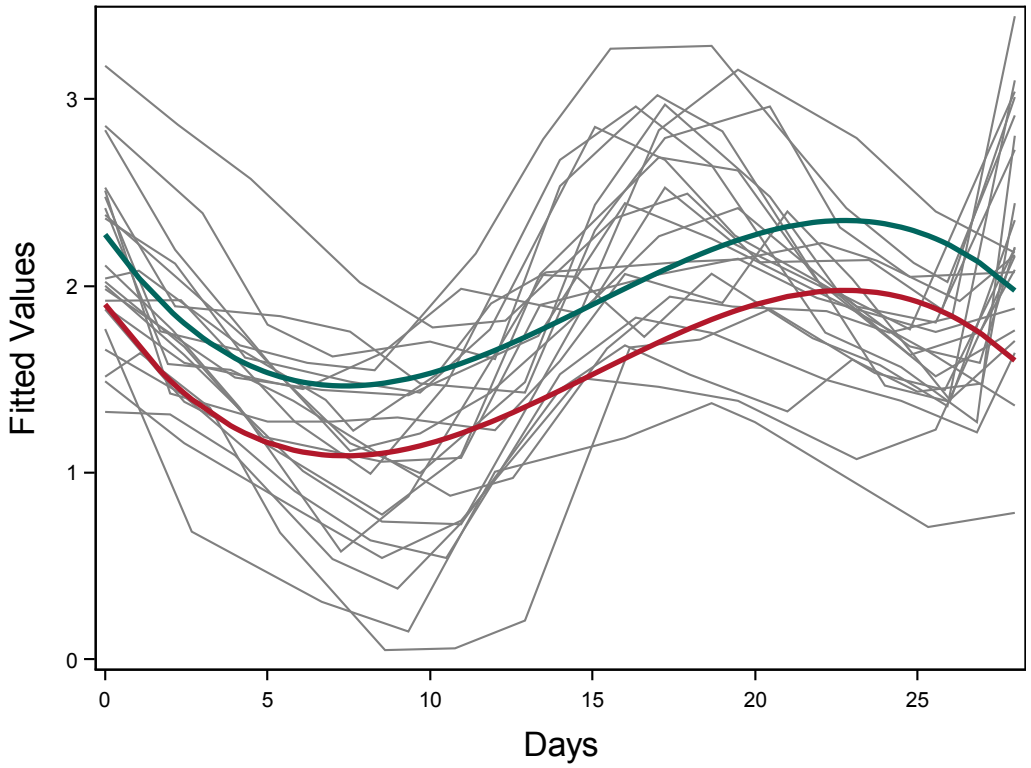
where $\delta = 1$ if $x_i > c$ and $\delta = 0$ if $x_i <= c$.

Cubic-Splines Modeling Approach (Gurrin et al., 2005)

$$u(x;\theta) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \sum_{i=1}^{m} \theta_i (x - c_i)_+^3$$

OH No! This is not what I wanted. (Problems?)

3rd degree polynomial regression



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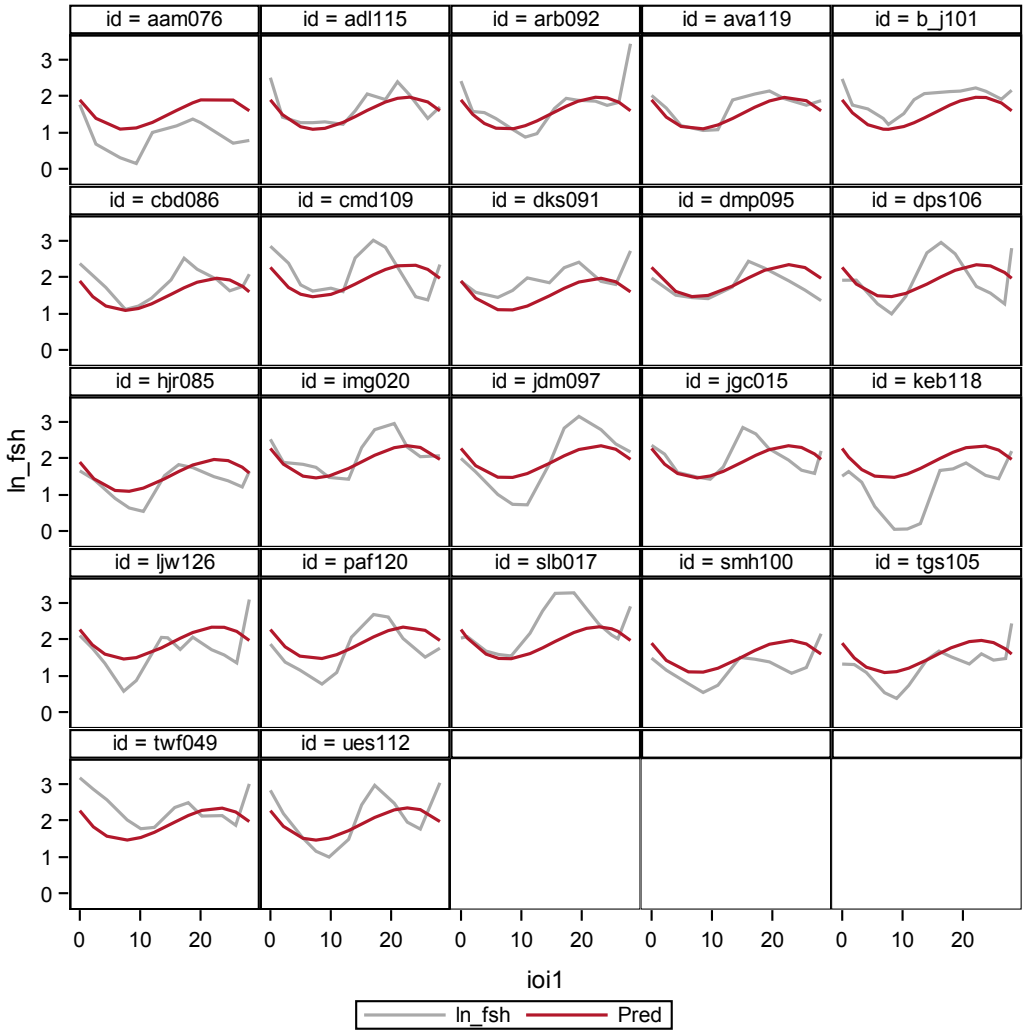

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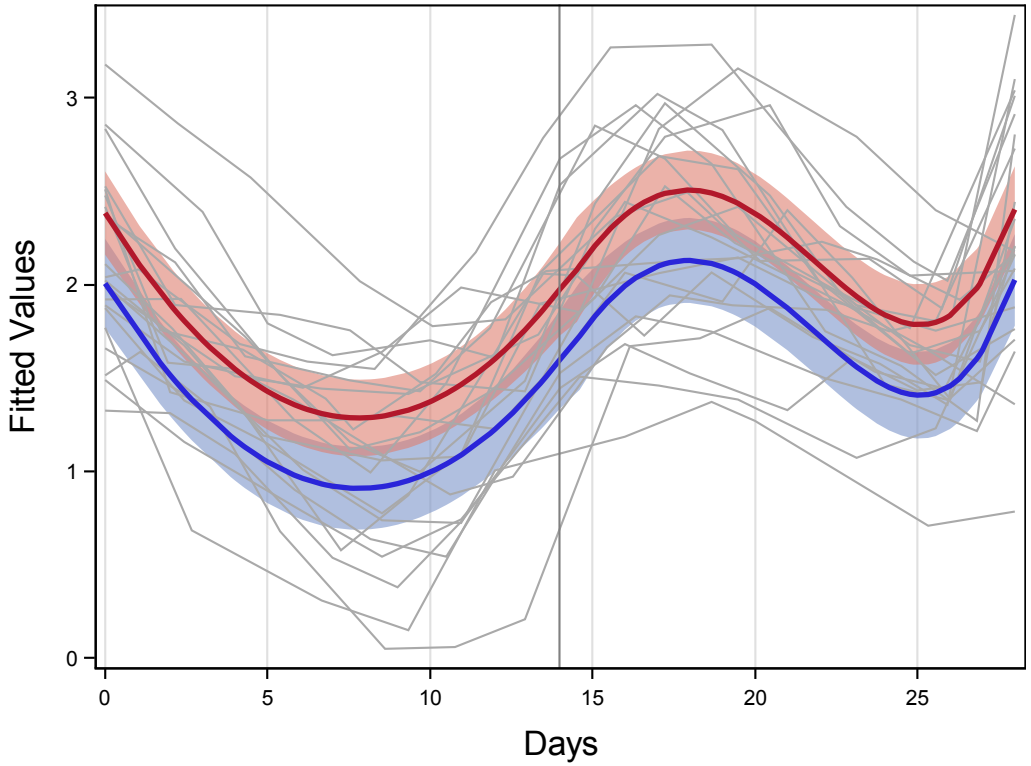
Piecewise-4th degree polynomial regression



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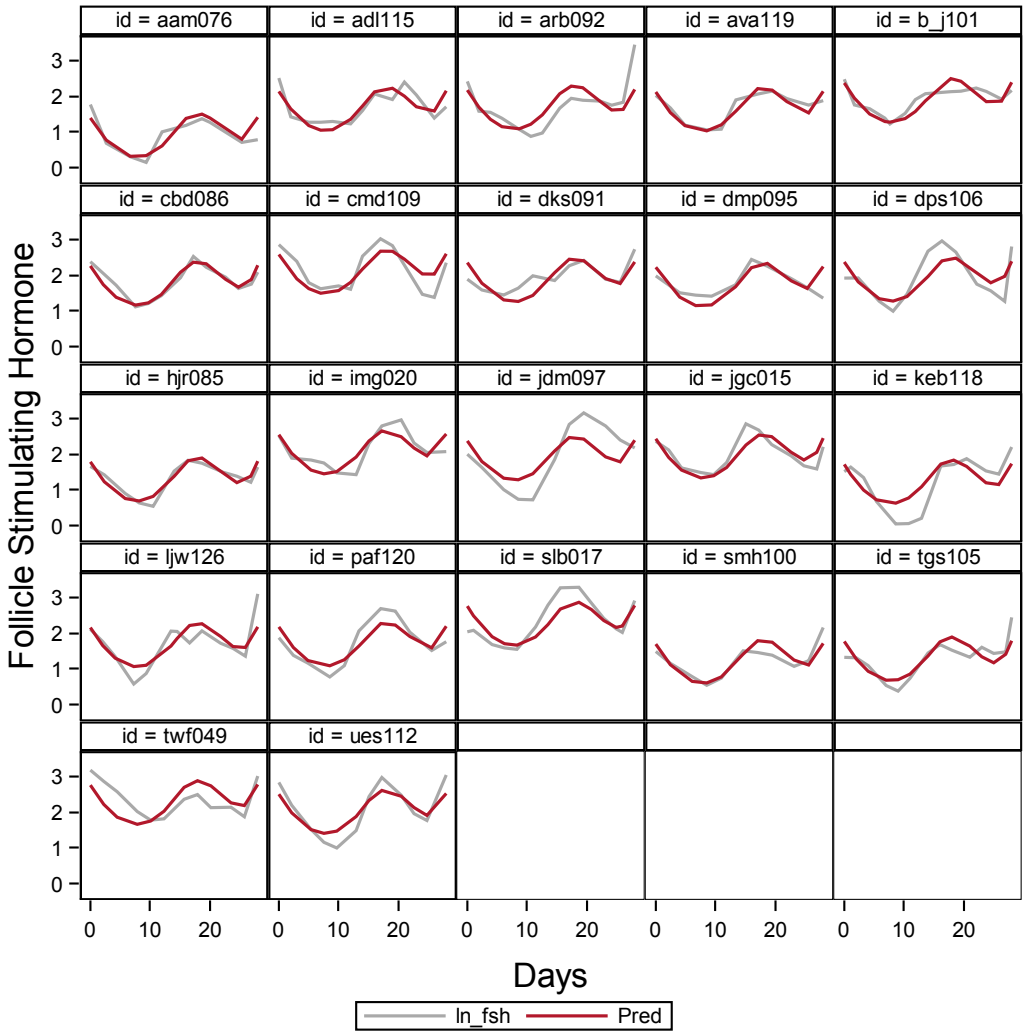

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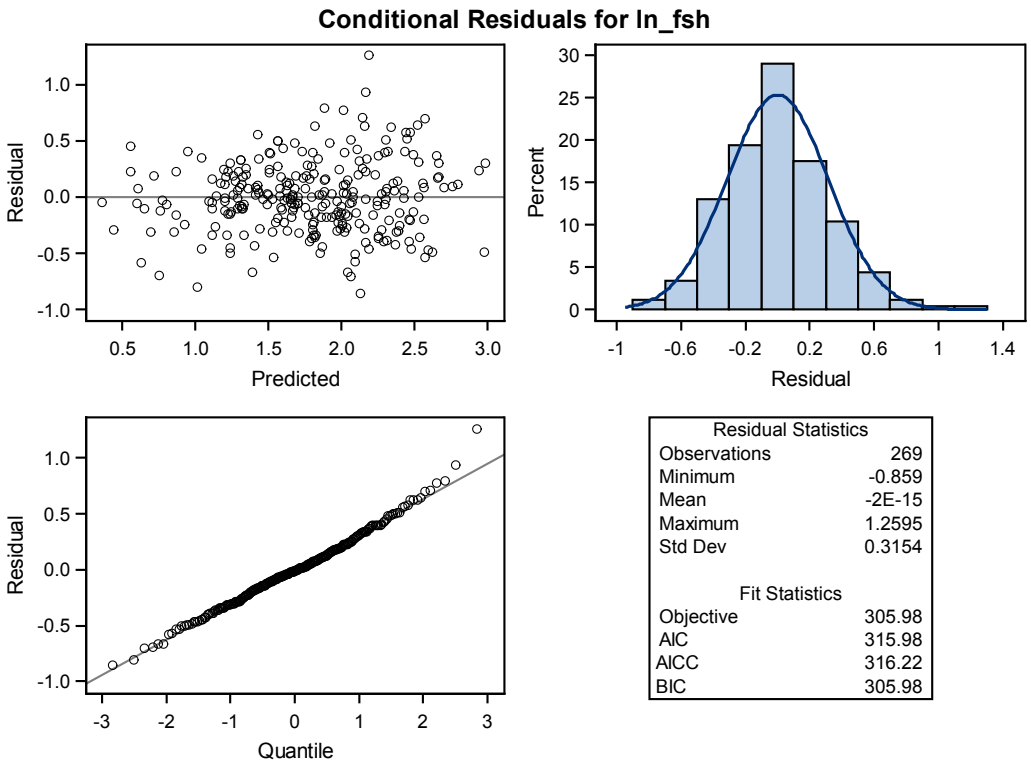
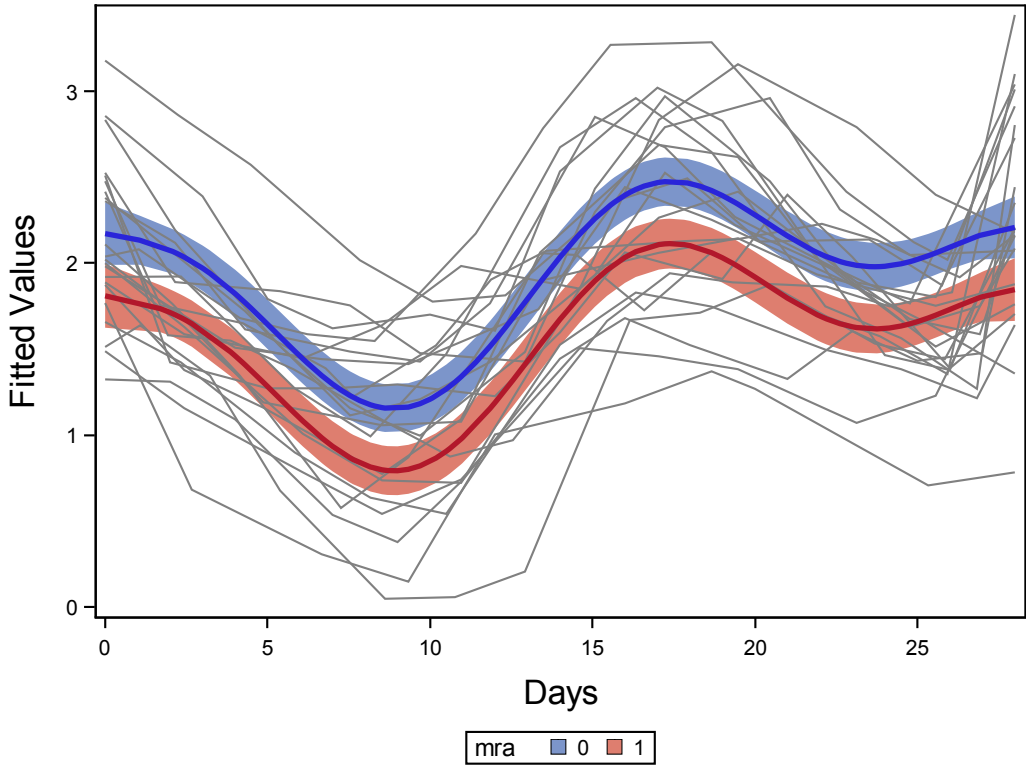


Residual Statistics	
Observations	269
Minimum	-0.859
Mean	-2E-15
Maximum	1.2595
Std Dev	0.3154
Fit Statistics	
Objective	305.98
AIC	315.98
AICC	316.22
BIC	305.98

Let's do more work! Still, ... Disadvantages???

3rd degree polynomial regression with trigonometric functions



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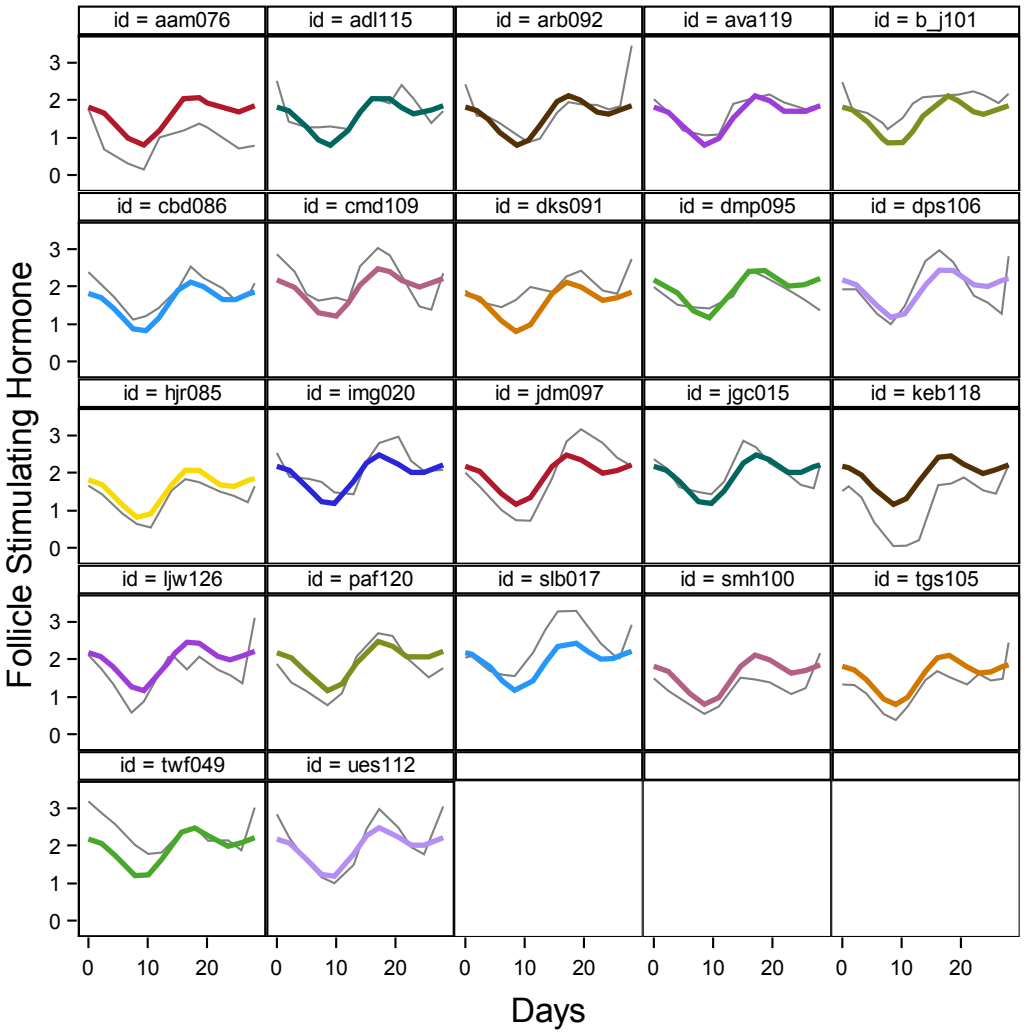
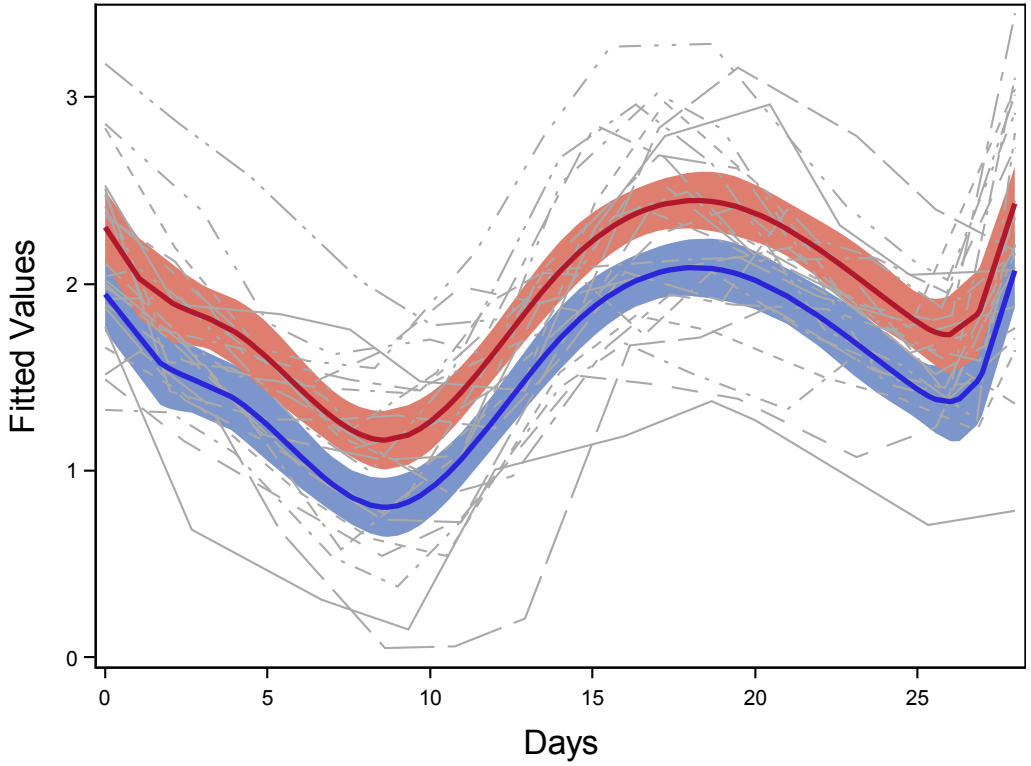


Much Better Now! Disadvantages???

Natural Cubic Splines



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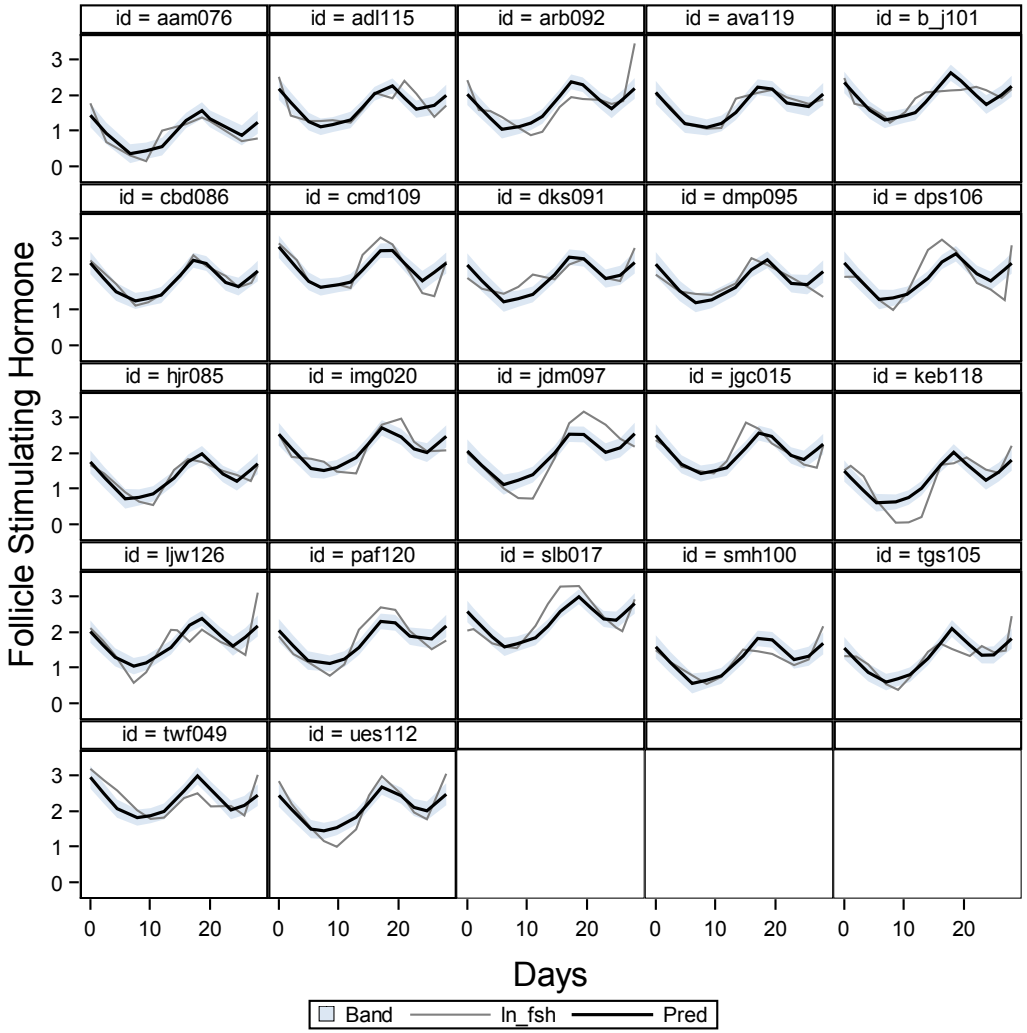

Questions

QUESTIONS?

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