

# Adjusting for Bias in Observational Data

Inverse Probability of Treatment  
Weighting using the Propensity  
Score



# Why Use Observational Data?

- Full spectrum of patients and providers.
- Some studies are not ethical to conduct as randomized trials.
- Much less expensive than randomized trials.
- Larger sample size, much longer follow-up for less common outcomes and long-term outcomes.



# But

- Assignment bias
- Missing information (variables that are not available)



# Dealing with Bias

- Regression
- Stratification
- Instrumental variable analysis
- Propensity score methods



# Calculating the Propensity Score

- A way of summarizing the information in all of the prognostic variables
- PS = probability of one of the two treatments, given the observed covariates
- Logistic regression:  
 $P(\text{treatment A rather than B}) = f(\text{age, sex, comorbidities, etc.})$

Propensity  
score



Logistic regression estimates the propensity for patients to be treated with A rather than B, based on patient characteristics

- `proc logistic descending;`
- `model A = age sex diabetes COPD`
- `rurality ...;`
- `output out = propensity predicted = PS;`
- PS ~ propensity of physicians to choose one treatment based on patient characteristics
- Patients predicted to be unlikely to be treated with A → low propensity score
- Patients predicted to be likely to be treated with A → high propensity score.



# Using the Propensity Score

- Stratification
- Regression
- Matching
- Inverse probability of treatment weighting





# Inverse Probability of Treatment Weighting Using the Propensity Score





# Weights

$$W = \frac{\textit{treatmentA}}{PS} + \frac{1 - \textit{treatmentA}}{1 - PS}$$

where  $\textit{treatmentA} = 1$  if the person received treatment A, and 0 if the person received treatment B



# Weights

Treatment A:  $W = \frac{1}{PS}$

Treatment B:  $W = \frac{1}{1 - PS}$

Subjects weighted by the inverse of the probability of receiving the treatment that was actually received.



# How it Works

- Create two datasets: one for each treatment group.
- Everyone contributes to both datasets



# Our Data

ID	Treatment group	PS P(A)	Outcome
1	A	0.80	$Y_1$
2	A	0.77	$Y_2$
3	B	0.70	$Y_3$
4	A	0.53	$Y_4$
5	B	0.50	$Y_5$
6	B	0.45	$Y_6$
7	A	0.33	$Y_7$
8	B	0.25	$Y_8$



# Data Set for the Effect of Treatment A

ID	Treatment group	PS P(A)	Outcome on Treatment A
1	A	0.80	$Y_1$
2	A	0.77	$Y_2$
3	B	0.70	?
4	A	0.53	$Y_4$
5	B	0.50	?
6	B	0.45	?
7	A	0.33	$Y_7$
8	B	0.25	?



# Data Set for the Effect of Treatment A

ID	Treatment group	PS P(A)	Weight = 1 / PS	Outcome on Treatment A
1	A	0.80	1.25	$Y_1$
2	A	0.77	1.30	$Y_2$
3	B	0.70	0	?
4	A	0.53	1.89	$Y_4$
5	B	0.50	0	?
6	B	0.45	0	?
7	A	0.33	3.03	$Y_7$
8	B	0.25	0	?

In dataset 1 we are missing information about the effect of treatment A for people who received B.

- If Mr. X, who received A, had a low (e.g. 20%) probability of getting A, there must be 4 similar people who received B.
- Mr. X's weight is  $1/0.2 = 5$ . He represents 5 people on treatment A (himself and 4 others).
- If they had received A, we expect their outcome would be the same as Mr. X's outcome. We impute the missing outcome for these people using Mr. X's outcome.





# Data Set for the Effect of Treatment B

ID	Treatment group	PS P(A)	Weight = $1 / (1 - PS)$	Outcome of Treatment B
1	A	0.80	0	?
2	A	0.77	0	?
3	B	0.70	3.33	$Y_3$
4	A	0.53	0	?
5	B	0.50	2.00	$Y_5$
6	B	0.45	1.82	$Y_6$
7	A	0.33	0	?
8	B	0.25	1.33	$Y_8$

# Estimating the Effects of Treatment A and Treatment B

Estimated average effect of treatment A

$$= \frac{1}{N} \sum_{i=1}^N \frac{\textit{treatmentA} \times Y_i}{PS}$$

Estimated average effect of treatment B

$$= \frac{1}{N} \sum_{i=1}^N \frac{(1 - \textit{treatmentA}) \times Y_i}{1 - PS}$$



# Treatment Difference

Estimated difference (treatment A – treatment B) =

$$\frac{1}{N} \sum_{i=1}^N \frac{(treatment) \times Y_i}{PS} -$$

$$\frac{1}{N} \sum_{i=1}^N \frac{(1 - treatment) \times Y_i}{1 - PS}$$

However, the estimate of the variance is not as straightforward.



# Interpretation

Result of an inversely weighted propensity score analysis is an aggregate estimate of the treatment effect, if it were applied to the entire population



- Unusual individuals (treated but don't fit the description of those usually treated,  $\therefore$  small PS) have high weights
- Unusual individuals (not treated, but look a lot like people who are usually treated,  $\therefore$  low value for  $(1-PS)$ ) have high weights
- May trim high weights.



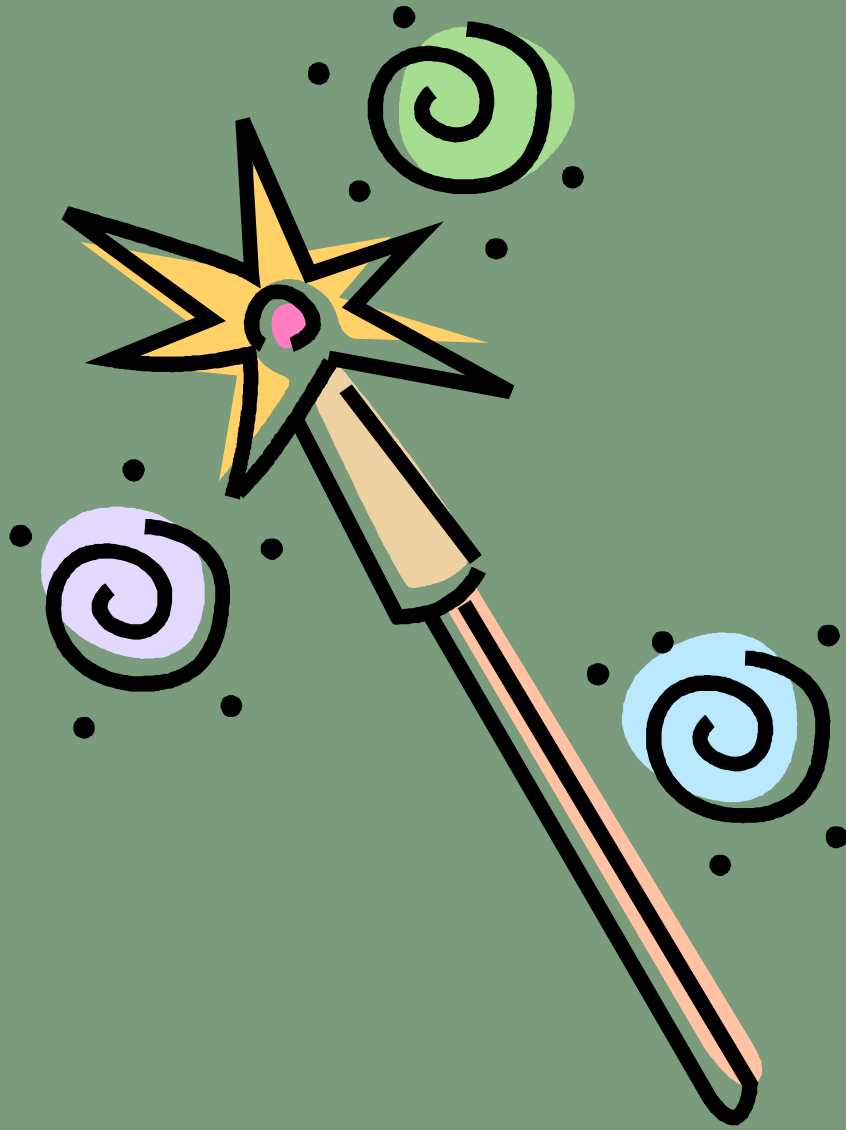
# IPW is Related to Survey Weights

In the CCHS, we oversample people from rural areas.

In order to obtain a population estimate, we upweight the responses from urban respondents and downweight the responses from rural people.



It's magic





# Well, almost magic

Makes no claims to balance unmeasured covariates.

Remove hidden biases only to the extent that unmeasured variables are correlated with the measures used to compute the score.



# What Questions are not Answered

- Does *not* predict the outcome for a person with a given set of characteristics
- Does *not* tell you the role of the other covariates in predicting the outcome (e.g. are older patients more likely to have a stroke) (this is what regression does).
- Does not tell you who will benefit most from a given treatment.



## References

Peter C. Austin. An introduction to propensity score methods for reducing the effects of confounding in observational studies. *Multivariate Behavioral Research*, 46:399–424, 2011

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