

The Spatial Autocorrelation Testing

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Abstract and goals:

Various evaluated cases exist in the space and are affected by their surrounding. This fact must be imported into the methods used for those cases evaluation. Most of environmental data has the basic characteristic in which the key role is not in the neighbourhood by administrative borders, but in their mutual distance. The aim of the article is to present the NBI index spatial autocorrelation testing using Mantel test. The Mantel statistics probability distribution is unknown. The adequate P-value is then necessary to compute using SAS aided simulations.

INTRODUCTION:

At the end of eightieth years, the international community began to discuss about global agreement which could cover in not only biodiversity protection but simultaneously also the task of approach to the genetic source, the long sustainable using of biodiversity and biotechnology, the creating of partnership between countries and the equal distribution of contributions from the genetic source using. Over the past ten years, the great progression was made in world-wide measure in the field of construction and the indicator evaluation of environmentally sustainable development. The World Bank, OSN and its following sections as UNDP, UNEP, WHO, FAO, World Resource Institute, World Watch Institute, and World Economic Forum on global level and Eurostat on European level devote oneself to the mentioned task. The key question for the most of environmental data is their mutual distance and no administrative bounds. The main aim of paper is the presentation of the evaluation of spatial autocorrelation NBI index by Mantel test which the only makes provision for object distance and no neighbourhood matrix i.e. the administrative bounds of territories.

MATERIALS AND METHODS

World Economic Forum (World Economic Forum, Yale Centre for Environmental Law and Policy) makes public National Biodiversity index. This index is based on subindices for number of variables belonging to four broad categories – housing, basic services, education, and health. These subindices for all variables are aggregated using equal weights to form the overall NBI index. It has to be kept in mind that the NBI suffers from serious, if not insurmountable, methodological weaknesses as there is no theoretical framework for aggregating indicators related to different dimensions of well-being, making any weighting scheme arbitrary. The index should therefore be interpreted very cautiously. The NBI index is just one from 76 variables of Environmental Sustainability Index (ESI). The ESI score represents an equally weighted average of the 21 indicator scores. Each indicator builds on between 2 and 12 data sets for a total of 76 underlying variables. Biodiversity, for example, is a composite indicator that includes variables tracking the national biodiversity index (Figure 1). The higher NBI value the better estimation of a state. Example of using Mantel's test is showed on NBI values from the year 2004.

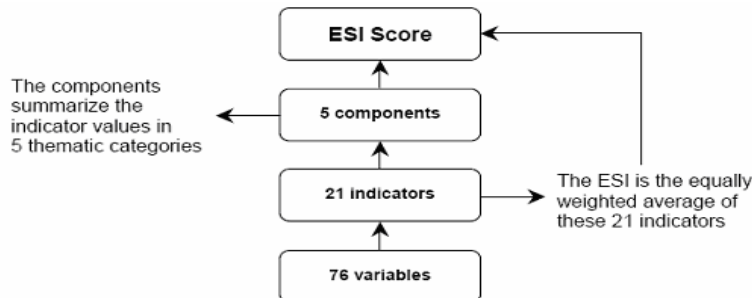


Figure 1: Constructing the ESI score

Source: <http://www.yale.edu/esi/>

Table 1 Values of NBI index

Country	NBI	Country	NBI	Country	NBI	Country	NBI
Belgium	0,45	Greece	0,55	France	0,42	Croatia	0,54
Ukraine	0,42	Italy	0,51	Germany	0,37	Latvia	0,42
Poland	0,37	Hungary	0,44	Slovenia	0,56	Austria	0,47
Romania	0,42	Slovakia	0,59	Denmark	0,40	Switzerland	0,50
Czech. Rep.	0,50	Belarus	0,37	Estonia	0,44	Sweden	0,30
Spain	0,49	Netherlands	0,41	Albania	0,53	Norway	0,30
Bulgaria	0,49	Portugal	0,37	Lithuania	0,42	Finland	0,29

Source: World Economic Forum

The **Mantel test** is a statistical test of the correlation between two matrices. The matrices must be of the same rank, and in the usual applications, they are matrices of interrelations between the same vectors of objects. The test is commonly used in ecology, where the data are usually estimates of the "distance" between objects such as species of organism. The test is available used for estimate of correlation between matrix of distance Q between objects from investigated attributes point of view and matrix of geographical distances of localities W . We can't just assess the relationship between the two matrices by evaluating the correlation coefficient between the two sets of distances and testing its statistical significance. The Mantel test deals with this problem. The procedure adopted is a kind of randomisation test. The correlation between the two sets of distances is calculated, and this is the measure of correlation reported. In principle any correlation coefficient could be used, but normally the Pearson product-moment correlation coefficient is used. The distances are calculated by arbitrary distance function, which is applicable for scale of sets. In our problem we used Euclidean distance in matrices W and Q .

Let's indicate q_{ij} ($i, j = 1, 2, \dots, m$) elements of matrix Q and let's indicate w_{ij} ($i, j = 1, 2, \dots, m$) elements of matrix W . Test statistics of Mantel test is

$$Z = \sum_{i=1}^m \sum_{j=1}^m q_{ij} w_{ij}$$

its standardize form is commonly used

$$r = \frac{1}{m^2 - m - 1} \sum_{i=1}^m \sum_{i=1}^m \frac{q_{ij} - \bar{q}}{s_q} \frac{w_{ij} - \bar{w}}{s_w}$$

where \bar{q} is average of values q_{ij} and s_q is standard deviation of values q_{ij} and \bar{w} is average of values w_{ij} , s_w is standard deviation of values w_{ij} . Value r gets values from interval $(-1, 1)$. Absolute values r are high if distances between elements of matrices Q and W are linear correlated. Probability distribution of Mantel statistics for small values m is not known. Standardized Mantel statistics has asymptotically normal distribution. Monte Carlo method is used to estimate significance of Mantel statistics. To assess significance, however, the rows and columns of one of the matrices are subjected to random permutations many times, with the correlation being recalculated after each permutation. The significance of the observed correlation is the proportion of such permutations that lead to a higher correlation coefficient. In the literature there is recommended for level of significance $\alpha = 0,05$ to use ordinal 1000 simulations and for level $\alpha = 0,01$ to use ordinal 10 000 simulations. P -value of Mantel statistics r we can calculate as following

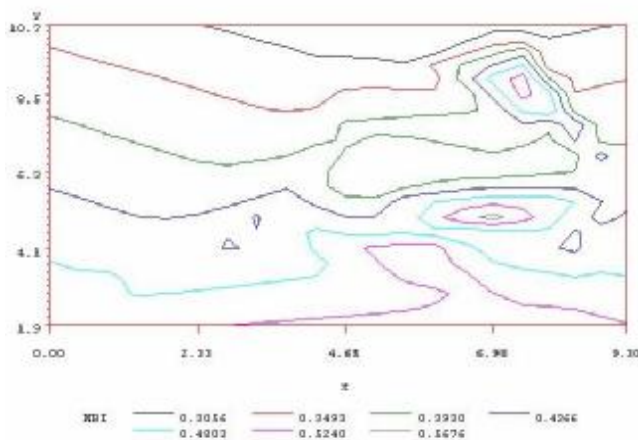
$$P = (\text{Number of simulations higher than testing statistics } r + 1) / (\text{number of simulations} + 1).$$

All the calculations were done using SAS software.

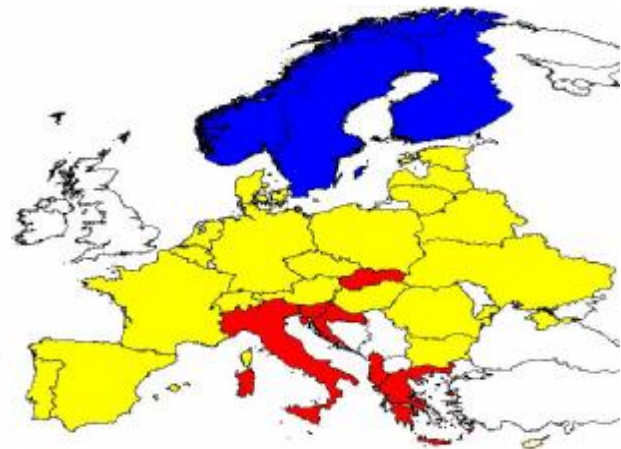
RESULTS

From environmental aspect of biodiversity evaluated by NBI on selected European countries, Finland, Norway and Sweden are in the worst situation. The NBI value for these countries is under value 0,30. Countries in the middle of Europe achieve very good NBI values (0,30 – 0,50). Mediterranean countries as Croatia, Italy, Albania, Slovenia show high NBI values (over 0,50) as well as Slovakia, which does not belong to Mediterranean countries (picture 1 and 2)

Picture 1 Graph of NBI



Picture 2



Source: Own calculation

Table 2 Results of Mantel test

Area	Pearson correlation coefficient r	Asymptotic probability	Empirical prob.	Number of simulations
European countries	0,2811	0,0014	0,0050	1 000
European countries	0,2811	0,0014	0,0040	1500
European countries	0,2811	0,0014	0,0048	5000
European countries	0,2811	0,0014	0,0040	10 000

Source: Own calculation

According to r statistics we can say there is spatial autocorrelation between European countries, it means, the countries geographically close to each other are alike in National biodiversity index. It is confirmed what many ecologists and environmentalists claimed long before: that biodiversity is larger in the countries with warmer climate than in the ones with colder climate. For example: Northern countries such as Sweden, Finland and Norway they have lower NBI value (0,30) than southern countries such as Italy (0,51) or Croatia (0,54).

Values of standardised statistics r of Mantel test for the countries chosen European countries are statistically confirmed on level of significance 0,05. Asymptotic P –values as well as empirical P -values achieved by Monte Carlo method with 1000 simulations are lower than 0,05. There were done 10 000 simulations. In this case P -value achieved (0,0040) is lower than level of significance 0,05. Values of r statistics are positive it means distance between areas and Mantel statistics values is direct linear correlation. Small area distances correlate with small differences in values. NBI index shows positive spatial autocorrelation.

CONCLUSION:

We used National Biodiversity index as an example of spatial autocorrelation evaluation of environmental sets. It does not come from administrative dividing of Europe. NBI positive spatial autocorrelations confirms the fact that from environmental aspect there are areas with higher biodiversity as well as the areas with lower biodiversity in Europe. It is influenced by geographical position of these countries and there is not a gap in biodiversity but it changes continuously.

Let's highlight another Mantel statistics benefit. Mantel test is the test of spatial correlation (Stehlíková, 2002). Other spatial autocorrelation tests such as Geary and Moran test they can be used just for one studied attribute. It is possible to use Mantel test for more attributes, and they don't have to be all quantitative. Gower distance using makes possible to define objects by quantitative and qualitative at once. Because of this reason Mantel test can be successively used in spatial autocorrelation evaluation of multiextended characteristics.

Used SAS modules and tools:

SAS/IML SAS/STAT

Data resources:

World Economic Forum

References:

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