

A background image featuring a man in a white shirt and blue tie standing in a field. The scene is overlaid with various digital and data-related graphics, including binary code (0s and 1s), mathematical symbols like $E=mc^2$, a globe, and a bar chart. A large, glowing yellow ring is also visible in the center of the image.

Introduction to the GLIMMIX Procedure

Saskatoon SAS Users Group

May 24, 2006

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Outline

- I. Introduction to Generalized Linear Mixed Models
- II. PROC GLIMMIX syntax
- III. An example

Special Thanks to and Complex Questions:

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I. Introduction to Generalized Linear Mixed Models

$$g(\mu) = X\beta + Z\gamma$$

When would it be used?

- Designed experiment with 2 drug treatment varieties and 8 random clinical sites
- Beta distribution

- PROC REG/GLM? Normal data, fixed effects
- PROC GENMOD? Fixed effects, beta not available
- PROC MIXED? Normal data
- PROC NLMIXED? More complex alternative

Generalized Linear Models (GLM)

Link function

Fixed effects

$$g(\mu) = X\beta$$

Design matrix for
fixed effects

$$\mu \equiv E(y)$$

$y \sim \text{exponential family}$

$$E(y) = g^{-1}(X\beta) \quad V(y) = V(\mu)a(\phi)$$

Linear Mixed Models (LMM)

Random effects

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\gamma} + \boldsymbol{\varepsilon}$$

Design matrix for
random effects

$$\boldsymbol{\gamma} \sim N(\mathbf{0}, \mathbf{G}), \quad \boldsymbol{\varepsilon} \sim N(\mathbf{0}, \mathbf{R})$$

$$E(\mathbf{y}) = \mathbf{X}\boldsymbol{\beta}, \quad \text{Var}(\mathbf{y}) = \mathbf{Z}\mathbf{G}\mathbf{Z}' + \mathbf{R} = \mathbf{V}$$

Generalized Linear Mixed Models (GLMM)

$$g(\mu) = X\beta + Z\gamma$$

$$\mu \equiv E(y | \gamma)$$

$y | \gamma \sim \text{exponential family}$

$$E(y | \gamma) = g^{-1}(X\beta + Z\gamma)$$

$$V(\gamma) = G \quad V(y) = A_{\mu}^{\frac{1}{2}} R A_{\mu}^{\frac{1}{2}}$$

Examples of GLMMs

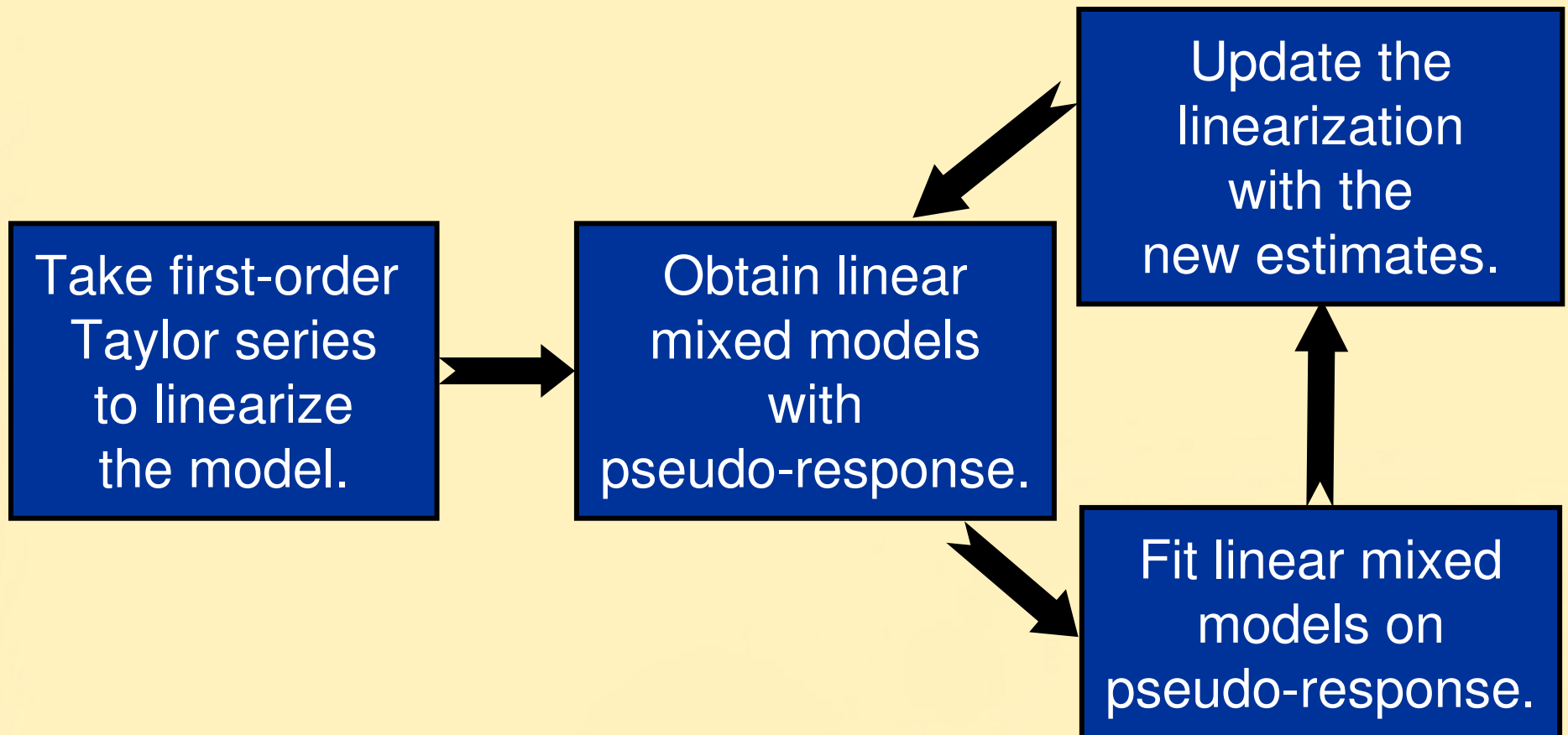
Responses	Conditional Distribution	Link	Variance	Model
Continuous	Normal	μ	σ^2	$\mu \gamma = X\beta + Z\gamma$
Binary	Binomial	$\log[\mu/(1-\mu)]$	$\mu(1-\mu)/n$	$\log[\mu/(1-\mu) \gamma] = X\beta + Z\gamma$
Count	Poisson	$\log(\mu)$	μ	$\log(\mu \gamma) = X\beta + Z\gamma$
Count	Negative Binomial	$\log(\mu)$	$\mu + k\mu^2$	$\log(\mu \gamma) = X\beta + Z\gamma$

Challenges in Fitting GLMMs

How can you obtain the marginal log-likelihood function?

- Linear mixed models:
 - Conditional distribution: $y|\gamma \sim N(X\beta + Z\gamma, R)$
 - Marginal distribution: $y \sim N(X\beta, V)$
- Generalized linear mixed models:
 - Conditional distribution: $y|\gamma \sim$ exponential family
 - Marginal distribution: *unknown*

Pseudo-Likelihood (Linearization) Method



Pseudo-Likelihood Method

- Four pseudo-likelihood estimation techniques:
 - RSPL Residual Subject-specific Pseudo-Likelihood
 - MSPL Maximum Subject-specific Pseudo-Likelihood
 - RMPL Residual Marginal Pseudo-Likelihood
 - MMPL Maximum Marginal Pseudo-Likelihood
- RxPL is less biased than MxPL because it profiles the fixed effects.
- xSPL models produce BLUPs.
- For linear mixed models, $RxPL=REML$ and $MxPL=ML$.

II. PROC GLIMMIX Syntax

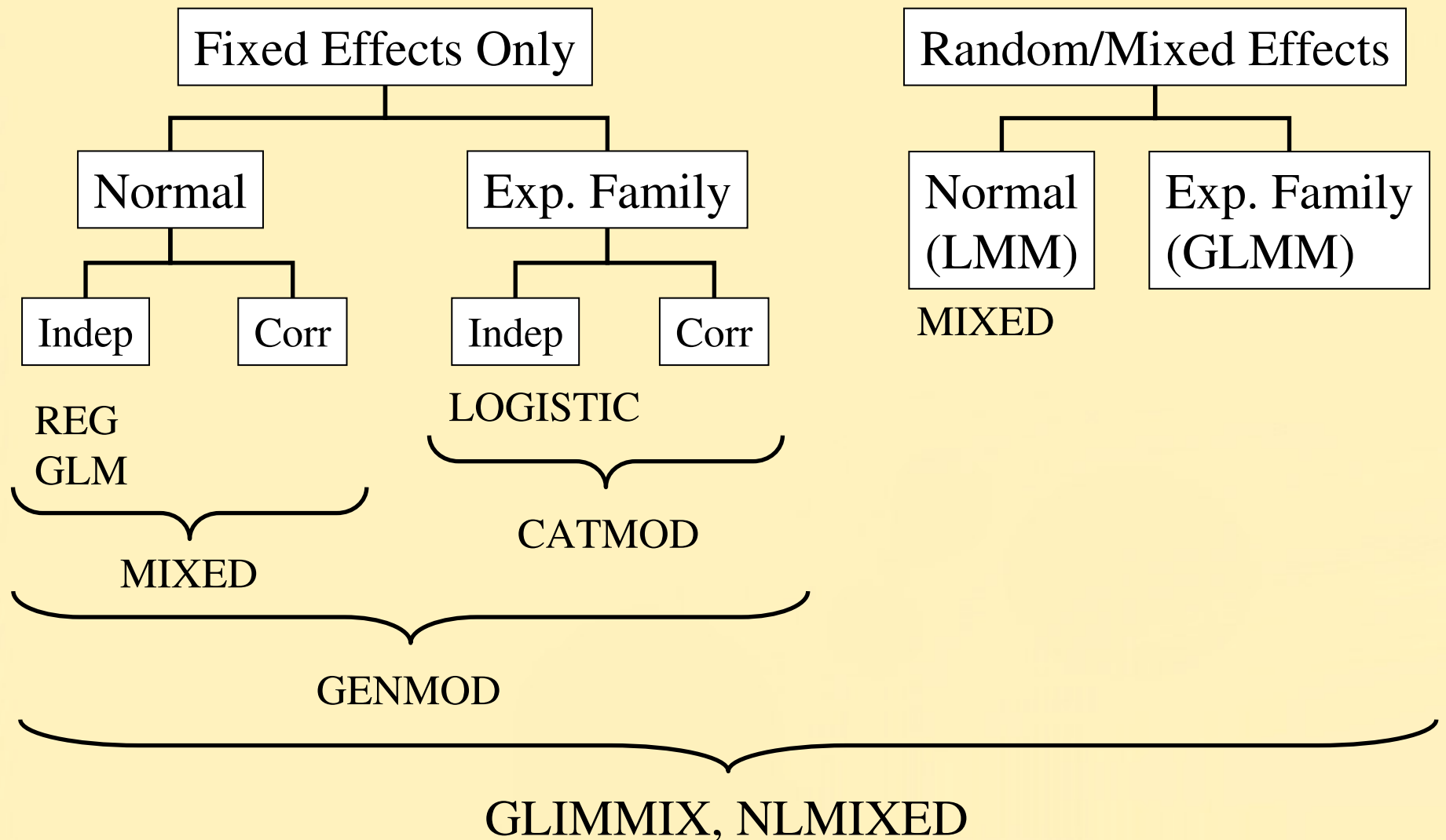
```
PROC GLIMMIX options ;  
    ...programming statements...;  
CLASS variables;  
MODEL response=fixed-effects / dist= link= options;  
RANDOM random-effects / options;  
RANDOM _residual_ / options;  
RUN;
```

The GLIMMIX Procedure

The GLIMMIX procedure is

- a new procedure in SAS/STAT for estimation and inference in generalized linear mixed models
- an add-on to SAS 9.1 for Windows only
- available by download from support.sas.com.

SAS Procedure Hierarchy for Linear Models



Generalized Linear Mixed Models (GLMM)

LINK= option

RANDOM statement

$$g(\mu) = X\beta + Z\gamma$$

MODEL statement

DIST= option

$y | \gamma \sim \text{exponential family}$

RANDOM statement
TYPE= option

RANDOM_RESIDUAL_
statement

$$V(\gamma) = G \quad V(y) = A_{\mu}^{\frac{1}{2}} R A_{\mu}^{\frac{1}{2}}$$

GLIMMIX Procedure

```
PROC GLIMMIX <options>;  
  CLASS variables;  
  CONTRAST 'label' contrast-specification </options>;  
  ESTIMATE 'label' contrast-specification </options>;  
  LSMEANS fixed-effects </options>;  
  LSMESTIMATE fixed-effect <'label'> values </options>;  
  MODEL response<(response options)>=<fixed-effects>  
    </options>;  
  NLOPTIONS <options>;  
  OUTPUT <OUT=SAS-data-set><keyword<(keyword-options) >  
    <=name>>.. </ options>;  
  PARMS (value-list)...</options>;  
  RANDOM random-effects </options>;  
  WEIGHT variable;  
RUN;
```

GLIMMIX Procedure

```
PROC GLIMMIX <options>;  
  CLASS variables;  
  CONTRAST 'label' contrast-specification </options>;  
  ESTIMATE 'label' contrast-specification </options>;  
  MODEL response = fixed-effects / DIST=  
  LINK= ;  
  <options>;  
  NLOPTIONS <options>;  
  OUTPUT <OUT=SAS-data-set><keyword<(keyword-options) >  
    <=name>>.. </ options>;  
  PARMS (value-list)...</options>;  
  RANDOM random-effects </options>;  
  WEIGHT variable;  
RUN;
```

Distributions Supported by PROC GLIMMIX

Discrete Response	Continuous Response
Binary	Beta
Binomial	Normal
Poisson	“Lognormal”
Geometric	Gamma
Negative Binomial	Exponential
Multinomial (nominal and ordinal)	Inverse Gaussian
+ combinations	Shifted T
	+ combinations

Link Functions in PROC GLIMMIX

- Cumulative Complementary log-log
- Cumulative logit
- Cumulative log-log
- Cumulative probit
- Complementary log-log
- Generalized logit
- Identity
- Log
- Logit
- Log-log
- Probit
- Power with exponent
- Power with exponent -2
- Reciprocal

GLIMMIX Procedure

```
PROC GLIMMIX <options>;  
  CLASS variables;  
  CONTRAST 'label' contrast-specification </options>;  
  ESTIMATE 'label' contrast-specification </options>;  
  LSMEANS fixed-effects </options>;  
RANDOM random-effects / TYPE = ;  
RANDOM _RESIDUAL_ / TYPE = ;  
  OUTPUT <OUT=SAS-data-set><keyword<(keyword-options) >  
    <=name>;  
  PARMS (value-list) </options>;  
  RANDOM random-effects </options>;  
  WEIGHT variable;  
RUN;
```

G-side

R-side

Selected COV Structures in PROC GLIMMIX

- Variance Components
- Compound Symmetry
- Unstructured
- First-Order Autoregressive
- Toeplitz
- Spatial Power
- Spatial Exponential
- Spatial Gaussian
- First-Order Factor Analytic
- Radial Smoother

GLIMMIX Procedure

```
PROC GLIMMIX <options>;  
  CLASS variables;  
  CONTRAST 'label' contrast-specification </options>;  
  ESTIMATE 'label' contrast-specification </options>;  
LSMESTIMATE fixed-effect 'label' values /  
options ;  
  </options>;  
  NLOPTIONS <options>;  
  OUTPUT <OUT=SAS-data-set><keyword<(keyword-options) >  
    <=name>>.. </ options>;  
  PARMS (value-list)...</options>;  
  RANDOM random-effects </options>;  
  WEIGHT variable;  
RUN;
```

III. An Example: Binomial Data in a Multi-center Clinical Trail

- 2 treatment methods, 8 clinic locations
 - Predict probability of a favorable response.
 - Clinics sampled from population and are thus random effects
-
- | | |
|-----------------|---------------------------------|
| • PROC REG/GLM? | Binary Distribution, non fixed |
| • PROC GENMOD? | Designed for fixed effects only |
| • PROC MIXED? | Binary Distribution |
| • PROC NLMIXED? | Possible alternative |



Questions?

Please accept an “I will find out”