Authentic Discourse in the Mathematics Classroom

Grayling B. Williams, Secondary Mathematics Specialist
Curriculum, Instruction, & Assessment
Durham Public Schools
Triangle High 5 Math Summit
SAS Institute
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Focusing on Communication to Make Mathematics Learning More Meaningful for Students

The Goals for Our Time Together

- To explore and articulate how the stage is already set for purposeful discourse in our mathematics classes
- To experience the power of communication in learning and understanding mathematics
- To validate the learner in ourselves so that we can access the learner in our students
- To have a good time while learning
What does the research reveal about struggling students?

- They bring real but unreliable knowledge to the mathematics classroom.
- They often lack the tools to recognize errors as they make them.
- They do not have the “habits and disposition” of good mathematics students.
- “Fixing” students with more initial teaching does not work.
What is Mathematical Discourse?

How would you define mathematical discourse?

To help us find words to define this idea, you have received a card with a common mathematical misconception.

Consider it alone for a moment, making notes on an index card.

Talk for a moment with a fellow participant.

Be prepared to share with the whole group.

Now, let’s define this notion as a group.

What is discourse?

It is defined as the interchange of ideas…especially conversation!

Positive Influences of Math Discourse

- Talk can reveal understanding and misunderstanding.
- Talk supports robust learning by boosting memory.
- Talk supports deeper reasoning.
- Talk supports language development.
- Talk supports the development of social skills.
How do students develop misconceptions?

- Misconceptions are NOT due to...
  - “Not listening”
  - Forgetting
  - Lack of motivation
- Students are learning what we are teaching them.
- The problem many times is the curriculum or how it is interpreted:
  - What is being taught
  - How it is being taught

Misconceptions often result from over-generalizing.

- Misconceptions often result from the misapplication of rules or concepts.
  - For example:
    - Lining up digits when adding
    - When multiplying two numbers, the product will be a larger number.

How do we fix misconceptions?

Two views:
- Pre-emptive teaching
- Encourage and plan for them to happen
How do we fix misconceptions?

- Create opportunities for misconceptions to arise.
- Place the student in “cognitive conflict”.
- Don’t “hide” the misconception; rather, bring it to light with discussion.

Implications for teaching

- Build on prior knowledge.
- Expose and discuss common misconceptions.
- Develop effective questioning.
- Use partners and small groups effectively.
- Emphasize methods rather than answers.
- Use rich, collaborative tasks.
- Create connections between closely related concepts.
- Use technology in appropriate ways.
  - Dr. Malcolm Swan, Shell Centre
Implications for teaching

- How is your statement connected to other statements?
- Staying with the conversation and asking questions
- Is it true?
- Is it the same?
- How did you do it?
- Huh? Taking action when you’re confused
- Use questions that put the mathematics to work.

We Remember:

- 20% of what we hear
- 50% of what we see and hear
- 70% of what we see, hear, and do
- 90% of what we see, hear, do, and tell about as we are doing it

The NCTM Process Standards

- Problem-Solving
- Reasoning and Proof
- Communication
- Connections
- Representation
CCSS Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Five Talk Moves

- Wait time
- Revoicing
- Restate
- Prompting
- Agree or Disagree

Chapin, O'Connor, & Anderson (2009)

Functions of the Talk Moves

- Using wait time
  - Allows students time to think.
  - Minimizes students’ tendencies to reason hastily.
  - Increases opportunities for equitable participation.
Functions of the Talk Moves

- Revoicing a student’s contribution provides opportunities for
  - Amplification.
  - Elaboration or increased clarity of reasoning.
  - The use of more mathematical language.
  - The set up of alignments and oppositions.
  - Attention and concern for student thinking and voice.

Functions of the Talk Moves

- Asking a student to restate
  - Builds a community of active listeners.
  - Provides another phrasing of reasoning for students to hear another opportunity to understand.
  - Is minute-by-minute formative assessment.

Functions of the Talk Moves

- Prompting for further participation
  - Increases opportunities for participation from a variety of students.
  - Gets multiple solutions/ideas on the table.
  - Pushes students to deeper levels of mathematical thinking.
Functions of the Talk Moves

- Asking students to apply their own reasoning to someone else’s reasoning (agree or disagree)
  - Encourages students to engage with one another’s ideas.
  - Directs attention to reasoning rather than to answers.
  - Helps them to make mathematical connections.
  - Promotes community argument (debate) and justification.

Now, we will use some familiar content in very different ways to see what happens and whether it helps us in developing this inquiry!

The variables, a, b, and c, represent positive whole numbers.

More on Justification and Reasoning

Work in pairs, divide the card stock into three sections:

1. Make sure each card contains a diagram and label it with its number.
2. Make sure each card contains a question and label it with its number.
3. Make sure each card contains a solution and label it with its number.

Your partner should either agree or disagree with the explanation or challenge it.

Either way, the response should be supported.

The variables, a, b, and c, represent positive whole numbers.
Opening: Complete the table, identify the rule, and write the function.

<table>
<thead>
<tr>
<th>Input x</th>
<th>Output y</th>
<th>Input x</th>
<th>Output y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>24</td>
<td>30</td>
<td>24</td>
<td>14</td>
</tr>
<tr>
<td>-4</td>
<td>2</td>
<td>-4</td>
<td>14</td>
</tr>
<tr>
<td>50</td>
<td>100</td>
<td>50</td>
<td>10</td>
</tr>
</tbody>
</table>

Card Sorting Activity

Work with teachers at your station or table to match the cards with their equivalent representations.

Card Sorting Activity

Presentations of Findings
MULTIPLE REPRESENTATIONS – LINEAR CARD SORT SOLUTIONS

<table>
<thead>
<tr>
<th>PINK (algebraic, graph)</th>
<th>GREEN (table)</th>
<th>YELLOW (graph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>F</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>E</td>
</tr>
<tr>
<td>C</td>
<td>G</td>
<td>H</td>
</tr>
<tr>
<td>D</td>
<td>D</td>
<td>B</td>
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<tr>
<td>E</td>
<td>B</td>
<td>C</td>
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<td>F</td>
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<td>D</td>
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<td>G</td>
<td>I</td>
<td>G</td>
</tr>
<tr>
<td>H</td>
<td>E</td>
<td>A</td>
</tr>
<tr>
<td>I</td>
<td>K</td>
<td>J</td>
</tr>
<tr>
<td>J</td>
<td>L</td>
<td>I</td>
</tr>
<tr>
<td>K</td>
<td>J</td>
<td>L</td>
</tr>
</tbody>
</table>

Text-Based Seminar

- A team examines an issue or concept from an outside point of view by focusing on a specific article or excerpt from a book.
- Text-based seminars help build a culture of discourse in a school by creating a safe place for individuals to approach issues or concepts.
- Participants read the article or excerpt from a book that is related to teaching and learning and engage in a discussion about the text.

- The purpose of the discussion is not to persuade other group members of a particular point of view but to clarify, build upon, and enhance understanding of the actual text.
- Text-based seminars give participants opportunity to extract different meanings and ideas from a text and discuss important issues related to the text.
Text-Based Seminar

“Never Say Anything A Kid Can Say”

• Select the text.
• Read (review) the text (5 - 7 minutes).
  – Take notes, highlight important ideas, record questions the text raises.
• Begin the discourse (3 - 5 minutes).
  – Take turns reading aloud a sentence or two that has particular significance to you.
  – Facilitator may present a framing question to start the discussion.
• Discuss the text (10 - 15 minutes).
  – Guidelines
• Close the discussion (3 minutes).
  – Highlight the two or three main points of discussion.
  – Thank participants.

“Never Say Anything a Kid Can Say!”

Definition of a lecture:

“The transfer of information from the notes of the lecturer to the notes of the student without passing through the minds of either.”

Never Say Anything a Kid Can Say!

List of reminders I have accumulated:
1. Never say anything a kid can say!
2. Ask good questions.
3. Use more process questions than product questions.
4. Replace lectures with sets of questions.
5. Be patient.
6. Be non-judgmental about a response or comment.
7. Try not to repeat students’ answers.
8. “Is this the right answer?”
9. Participation is not optional!
Never Say Anything a Kid Can Say!

List of techniques:
1. Use the think-pair-share strategy.
2. Always require students to ask a question when they need help.
3. Require several responses to the same question.
4. No one in a group is finished until everyone in the group can explain and defend the solution.
5. Use hand signals often.
7. Avoid answering my own questions.
8. Ask questions of the whole group.
9. Do not allow students to blurt out answers.

My definition of a good teacher has since changed from: "one who explains things so well that students understand" TO "one who gets students to explain things so well that they can be understood."

Debriefing the Activities: Do activities like this one help us prove the point?

Having trouble getting started? Consider using...

- A Graph
- A Table
- An Equation
- A Picture
- A Number Line

For example:

<table>
<thead>
<tr>
<th>Original Price</th>
<th>Sale Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>20% 20% 20% 20%</td>
<td>$5</td>
</tr>
</tbody>
</table>

Foxtrot


