A Performance-Based Assessment: A Means to High-Level Thinking and Reasoning

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Grade 4
Forms of Assessment

Assessment as Learning

Assessment of Learning

Assessment for Learning
Session Goals

• Deepen understanding of the Common Core State Standards (CCSS) for Mathematical Practice and Mathematical Content.

• Understand how Performance-Based Assessments (PBAs) assess the CCSS for both Mathematical Content and Practice.

• Understand the ways in which PBAs assess students’ conceptual understanding.
Overview of Activities

• Analyze and discuss the CCSS for Mathematical Content and Mathematical Practice.

• Analyze PBAs in order to determine the way the assessments are assessing the CCSSM.

• Discuss the CCSS related to the tasks and the implications for instruction and learning.

• Discuss what it means to develop and assess conceptual understanding.
The Common Core State Standards

The standards consist of:

- The CCSS for Mathematical Content
- The CCSS for Mathematical Practice
Analyzing a Performance-Based Assessment
North Carolina Focus Clusters
Grade 4  CHECK ON THESE

• Extend understanding of fraction equivalence and ordering.

• Build fractions from unit fractions by applying and extending previous understanding of operations of whole numbers.
Analyzing Assessment Items
(Private Think Time)

Four assessment items have been provided:
- Pizza Task
- More or Less Than One? Task
- 24 Cookies Task
- The Cake Shop Task

For each assessment item:
- Solve the assessment item.
- Make connections between the standard(s) and the assessment item.
1. Pizza Task

Both Harry and Jaden ate a portion of the same size pizza.

a. Show the portion of pizza that each student ate.

Harry’s pizza:

- Harry’s pizza is cut into 8 pieces, and he ate 3/8 of the pizza.

Jaden’s pizza:

- Jaden’s pizza is cut into 4 pieces, and she ate 1/4 of her pizza.

b. Use words and fractions to explain who ate the most pizza.

c. Refer to the pizzas and explain how much more pizza one person ate than the other person.
2. More or Less than One? Task

Below are two addition problems. Without doing the work for each of the problems below, make diagrams and explain how you know if the sum is more or less than one.

Problem A: 1/2 + 1/3 + 1/4 = _____

Problem B: 1/3 + 1/4 + 1/5 = _____
3. 24 Cookies Task

Three children are sharing 24 cookies.

Maria says, “I want 1/3 of the set of 24 cookies.”

Ted says, “I want 1/2 of the set of cookies.”

Monica says, “I want 1/4 of the set of cookies.”

Use diagrams and equations to explain if it is possible for Maria, Ted and Monica to each have the fraction of the cookies they want. Write about the relationship between the 24 cookies that are available and the fraction of cookies each student wants.
4. The Cake Shop Task

Lisa buys different kinds of cake when she goes to the bakery. She likes to get chocolate, vanilla, cherry, and almond cake.

a. Lisa buys 4 pieces of cake. Each piece is 2/6 of a cake. How much cake does she have altogether? Show a diagram and write an equation that shows and describes Lisa's cake.

Lisa claims:

I can figure out the number of pieces that I have by thinking about the pieces that are each 1/6 of the cake.

b. When Lisa thinks about the cake this way, she writes 4 x 2 x 1/6 of a cake. Explain why Lisa can think about the cake this way. Use words or diagrams.
Discussing Content Standards
(Small-Group Time)

For each assessment item:

With your small group, discuss the connections between the content standard(s) and the assessment item.
Deepening Understanding of the Content Standards via the Assessment Items (Whole Group)

As a result of looking at the assessment items, what do you better understand about the specifics of the content standards?

What are you still wondering about?
## Number and Operations – Fractions

### 4.NF

#### Extend understanding of fraction equivalence and ordering.

4.NF.1  
Explain why a fraction \( \frac{a}{b} \) is equivalent to a fraction \( \frac{(n \times a)}{(n \times b)} \) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. §

4.NF.2  
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

#### Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3  
Understand a fraction \( \frac{a}{b} \) with \( a > 1 \) as a sum of fractions \( \frac{1}{b} \).

4.NF.3a  
Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. ♦
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

4.NF.4a Understand a fraction \( \frac{a}{b} \) as a multiple of \( \frac{1}{b} \). For example, use a visual fraction model to represent \( \frac{5}{4} \) as the product \( 5 \times \frac{1}{4} \), recording the conclusion by the equation \( \frac{5}{4} = 5 \times \frac{1}{4} \).

4.NF.4b Understand a multiple of \( \frac{a}{b} \) as a multiple of \( \frac{1}{b} \), and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express \( 3 \times \frac{2}{5} \) as \( 6 \times \frac{1}{5} \), recognizing this product as \( \frac{6}{5} \). (In general, \( n \times \frac{a}{b} = \frac{n \times a}{b} \).)

4.NF.4c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat \( \frac{3}{8} \) of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?
Determining the Mathematical Practices Associated with the Performance-Based Assessment
Getting Familiar with the CCSS for Mathematical Practices
(Private Think Time)

• Count off by 8. Each person reads one of the CCSS for Mathematical Practices.

• Read your assigned Mathematical Practice. Be prepared to share the “gist” of the Mathematical Practice.
The CCSS for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Common Core State Standards for Mathematics, 2010, NGA Center/CCSSO
Discussing Practice Standards  
(Small-Group Time)

Each person has 2 minutes to share important information about his/her assigned Mathematical Practice.
Discussing Practice Standards
(Small-Group Time)

For each assessment item:

With your small group, discuss the connections between the practice standards and the assessment item.
Deepening Understanding of the Practice Standards via the Assessment Items

(Whole Group)

Which mathematical practices do you better understand?

What are you still wondering about?
The CCSS for Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Assessing Conceptual Understanding
Rationale

We have now examined assessment items and discussed their connection to the CCSS for Mathematical Content and Practice. A question that needs considering, however, is if and how these assessments will give us a good means of measuring the conceptual understandings our students have acquired.

In this activity, you will have an opportunity to consider what it means to develop conceptual understanding as described in the CCSS for Mathematics and what it takes to assess for it.
Assessing for Conceptual Understanding

The set of PBA items are designed to assess student understanding of fraction equivalence and ordering, and of building fractions from unit fractions.

Look across the set of related items. What might a teacher learn about a student’s understanding by looking at the student’s performance across the set of items as a whole?

*What is the variance from one item to the next?*
Developing and Assessing Understanding

Why is it important, when assessing a student’s conceptual understanding, to vary items in these ways?
Conceptual Understanding

• What do the authors mean by conceptual understanding?

• How might analyzing student performance on this set of assessments help us determine if students have a deep understanding of Number and Operations – Fractions?
Developing Conceptual Understanding

Knowledge that has been learned with understanding provides the basis of generating new knowledge and for solving new and unfamiliar problems. When students have acquired conceptual understanding in an area of mathematics, they see connections among concepts and procedures and can give arguments to explain why some facts are consequences of others. They gain confidence, which then provides a base from which they can move to another level of understanding.

The CCSS on Conceptual Understanding

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Content and the Standards for Mathematical Practice.

These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Common Core State Standards for Mathematics, 2010, p. 8, NGA Center/CCSSO
Assessing Concept Image

Tall (1992) differentiates between the mathematical definition of a concept and the concept image, which is the entire cognitive structure that a person has formed related to the concept. This concept image is made up of pictures, examples and non-examples, processes, and properties.

A strong concept image is a rich, integrated, mental representation that allows the student to flexibly move between multiple formulations and representations of an idea. A student who has connected mathematical ideas in this way can create and use a model to analyze a situation, uncover patterns and synthesize them to form an integrated picture. They can also use symbols meaningfully to describe generalizations which then provides a base from which they can move to another level of understanding.

Analyzing a Student’s Performance

Analyze Linda’s performance on four tasks.

What do you notice? What does Linda know?
1. Pizza Task: Linda’s Work

Both Harry and Jaden ate a portion of the same size pizza.

a. Show the portion of pizza that each student ate.

Harry’s pizza:

Harry’s pizza is cut into 8 pieces and he ate \( \frac{3}{8} \) of the pizza.

Jaden’s pizza:

Jaden’s pizza is cut into 4 pieces and she ate \( \frac{1}{4} \) of her pizza.

b. Use words and fractions to explain who ate the most pizza.

Harry has more than Jaden.

\( \frac{3}{8} > \frac{1}{4} \)

c. How much more pizza did one person eat than the other person?

Harry ate \( \frac{3}{8} \) pizza. This is more than Jaden.
2. More or Less than One? Task: Linda’s Work

Below are two addition problems. Make diagrams and explain how you know if the sum is more or less than one.

Problem A: \( \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \) _____

Problem B: \( \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \) _____
Three children are sharing 24 cookies.

Maria says, “I want 1/3 of the set of 24 cookies.”
Ted says, “I want 1/2 of the set of 24 cookies.”
Monica says, “I want 1/4 of the set of 24 cookies.”

Is it possible for Maria, Ted and Monica to each have the fraction of the cookies they want? _____

If you respond yes, use diagrams and equations to explain how you know they can each receive the share of cookies they want.

If you respond no, use diagrams and equations to explain why they cannot receive the number of cookies they each want.

The children of the household don’t all get Maria gets 8 cookies because 24 ÷ 3 = 8.
Ted gets 12 because 24 ÷ 2 = 12.
Monica wants 6 because 24 ÷ 4 = 6.

One person gets 2 cookies less than he/she wants or 2 people get 1 less cookie they want.
4. The Cake Shop Task: Linda's Work

Lisa buys different kinds of cake when she goes to the bakery. She likes to get chocolate, vanilla, cherry, and almond cake.

a. Lisa buys 4 pieces of cake. Each piece is 2/6 of a cake. How much cake does she have altogether? Show a diagram and write an equation that shows and describes Lisa's cake.

Lisa claims:

I can figure out the number of pieces that I have by thinking about the pieces that are each 1/6 of the cake.

b. When Lisa thinks about the cake this way, she writes 4 x 2 x 1/6 of a cake. Explain why Lisa can think about the cake this way. Use words or diagrams.

\[ 4 \times \frac{2}{6} = \frac{8}{6} \]
Using the Assessment to Think About Instruction

In order for students to perform well on the PBA, what are the implications for instruction?

• What kinds of instructional tasks will need to be used in the classroom?

• What will teaching and learning look like and sound like in the classroom?
Step Back

• What have you learned about the CCSS for Mathematical Content that surprised you?

• What is the difference between the CCSS for Mathematical Content and the CCSS for Mathematical Practices?

• Why do we say that students must work on both Mathematical Content and the Mathematical Practices?
Representations of Equivalent Fractions

Using an area model to show that $\frac{2}{3} = \frac{4 \times 2}{4 \times 3}$

The whole is the square, measured by area. On the left it is divided horizontally into 3 rectangles of equal area, and the shaded region is 2 of these and so represents $\frac{2}{3}$. On the right it is divided into $4 \times 3$ small rectangles of equal area, and the shaded area comprises $4 \times 2$ of these, and so it represents $\frac{4 \times 2}{4 \times 3}$.

Using the number line to show that $\frac{4}{3} = \frac{5 \times 4}{5 \times 3}$

$\frac{4}{3}$ is 4 parts when each part is $\frac{1}{3}$, and we want to see that this is also $5 \times 4$ parts when each part is $\frac{1}{5 \times 3}$. Divide each of the intervals of length $\frac{1}{3}$ into 5 parts of equal length. There are $5 \times 3$ parts of equal length in the unit interval, and $\frac{4}{3}$ is $5 \times 4$ of these. Therefore $\frac{4}{3} = \frac{5 \times 4}{5 \times 3} = \frac{20}{15}$.

Representations of Adding and Subtracting Fractions

1. Pizza Task

Number and Operations – Fractions

4.NF

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4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Common Core State Standards, 2010, NGA Center/CCSSO
2. More or Less than One? Task

Number and Operations – Fractions 4.NF

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Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.3a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

Common Core State Standards, 2010, NGA Center/CCSSO
3. 24 Cookies Task

Number and Operations – Fractions 4.NF

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.4b Understand a multiple of \( a/b \) as a multiple of \( 1/b \), and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express \( 3 \times (2/5) \) as \( 6 \times (1/5) \), recognizing this product as \( 6/5 \). (In general, \( n \times (a/b) = (n \times a)/b \).

4.NF.4c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat \( 3/8 \) of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?
4. The Cake Shop Task

Number and Operations – Fractions 4.NF

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4.NF.4a Understand a fraction \( a/b \) as a multiple of \( 1/b \). For example, use a visual fraction model to represent \( 5/4 \) as the product \( 5 \times (1/4) \), recording the conclusion by the equation \( 5/4 = 5 \times (1/4) \).

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Common Core State Standards, 2010, NGA Center/CCSSO