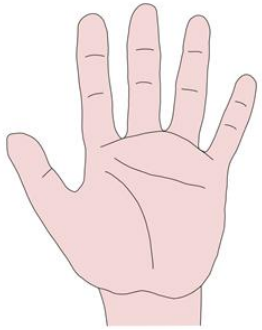
Welcome!



As you are settling in, please do the following:

- Trace or draw your hand on a blank sheet of paper from your table
- Fill in the following information on your hand:
  - **Palm** – your name
  - **Thumb** – grade level(s) you teach
  - **Index finger** – school name and county
  - **Middle finger** – one success you've had with Math Common Core Curriculum
  - **Ring finger** – one challenge you've had with Math Common Core Curriculum
  - **Pinky finger** – one thing you hope to get out of this session

http://cliparts101.com/free_clipart/24783/hand (graphic)

Time for a Video

Fractions: Teaching and Learning

What makes fractions so hard for students?

- Taught too abstractly with limited models
- Taught with rote memorization of procedures
- Not taught in meaningful contexts
- More attention to algorithms rather than to developing number sense and reasoning

Progression of Denominators

3rd Grade:	4th Grade:	5th Grade:
Fraction denominators 2, 3, 4, 6, & 8	Fraction denominators 2, 3, 4, 5, 6, 8, 10, 12, & 100	No restrictions on denominators

Critical Area- Fractions
(3rd grade)

Developing understanding of fractions, especially unit fractions (fractions with numerator 1).

Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.


fractions
number
whole
ribbon
equal
Recognize
understanding
a/b
divided
Represent
part
Explain
locates
two
1/3
understand
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unit
Understand
comparisons
recognize
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models
fraction
parts
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e.g
represent
Students
numbers
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Express
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Note
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6/1
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Grade
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conclusions
use
small
problems
3.NF.1
2/4
3.NF.3
partitioned
lengths
cases
beginning
able
locate
refer
generate
resulting
equivalence
3.NF.2
relative
domain
valid
4/6
denominator
especially
eveloping
expectations
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built
2/3
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Examples
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numerators
comparing
involve
view
along
solve
special
example
marking
partitioning
noticing
symbols
Develop
fractional
3/1
4/4
Compare
justify
general

Key Concepts about Fractions
(3rd grade)

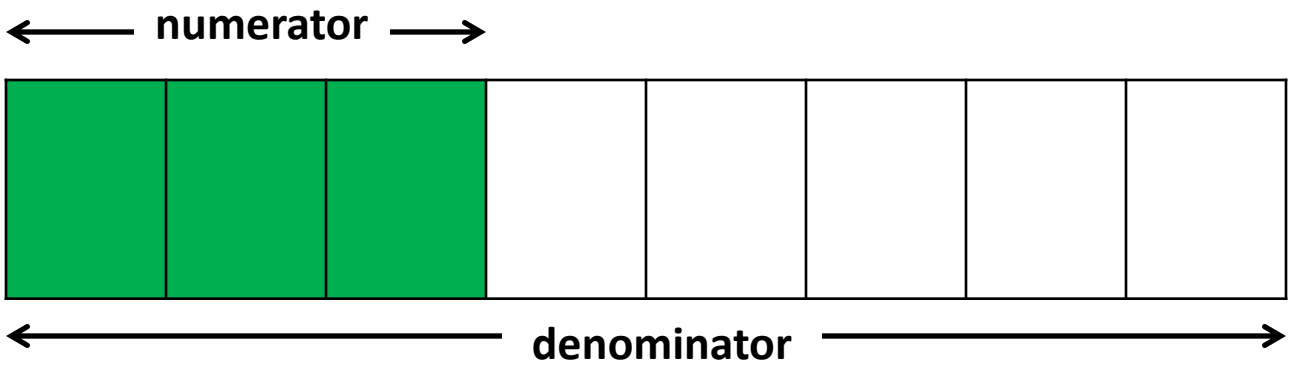
- Unit fractions
- Visual fraction models
- Compare fractions including equivalent fractions (<, >, =)
    - Denominators 2, 3, 4, 6, 8
    - Use number lines and models
    - Reason about size (same numerator or same denominator)

Numerators & Denominators

The numerator indicates how many fractional parts are under consideration.

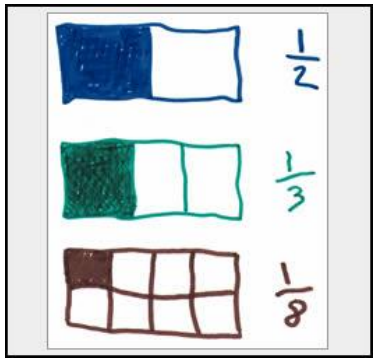
$$\frac{3}{8}$$

The denominator indicates by what number the whole has been partitioned (divided).

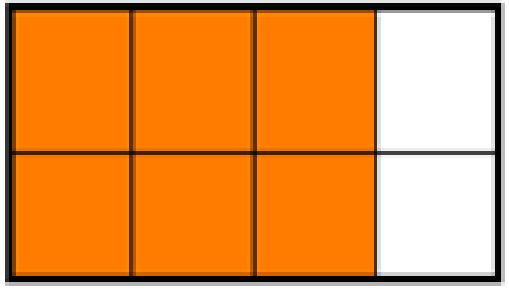


Unit Fractions

- One fractional part is known as a “unit fraction”
- A unit fraction is a single fractional part of a whole (1/2, 1/3, 1/4, 1/8, etc.)
- The more parts a whole is divided into, the smaller the parts



Exploring Unit Fractions



This fraction shows $\frac{6}{8}$

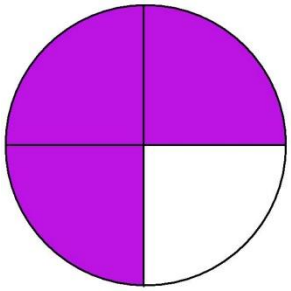
The fraction $\frac{6}{8}$ is made up of six, $\frac{1}{8}$ pieces

$$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{6}{8}$$

Practice with Unit Fractions

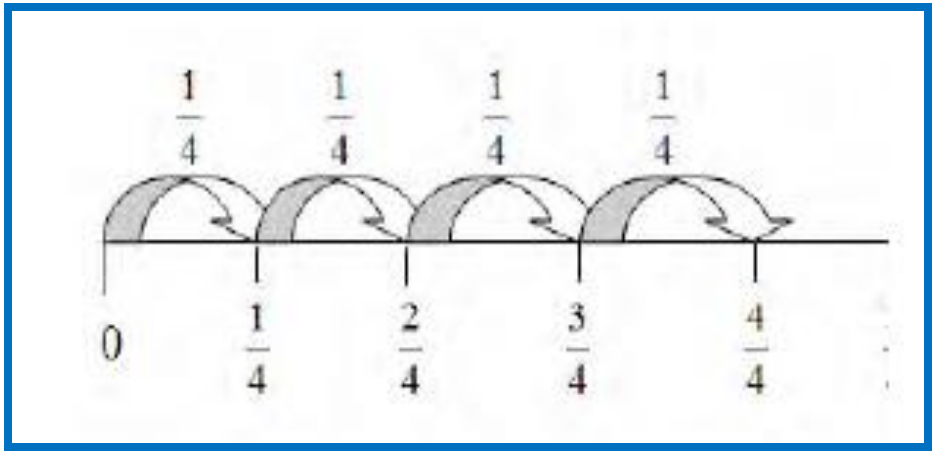
1. Select a fraction with a denominator of 2, 3, 4, 6 or 8.
2. Draw a model of your fraction on a piece of graph paper.
3. Draw a model that shows your fraction broken into unit fractions.
4. Write the numerical representation of your fraction.

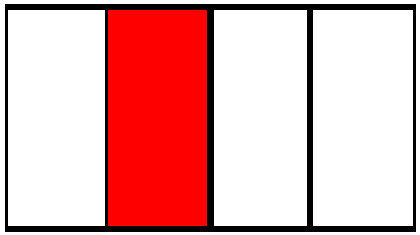
$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

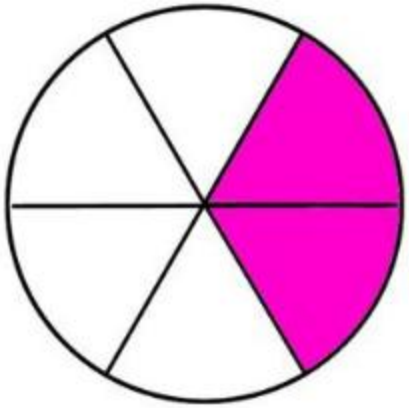


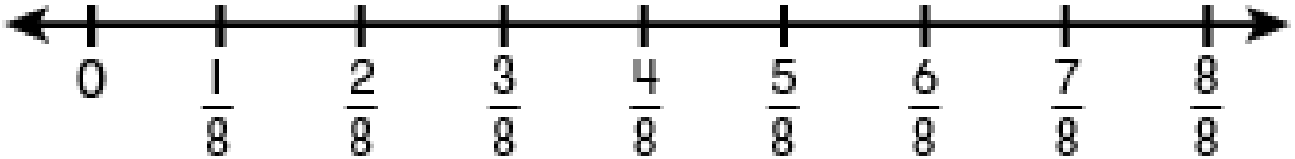
Visual Models

- Area Models (parts of a whole)
  - Including circles, rectangles & squares
  - **No** Set Models
- Number Lines



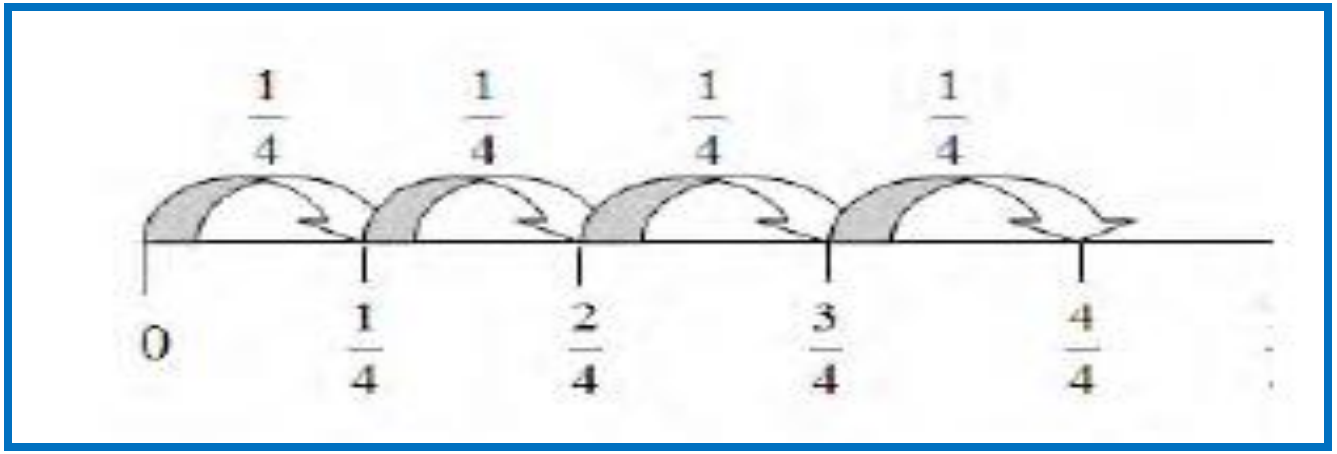






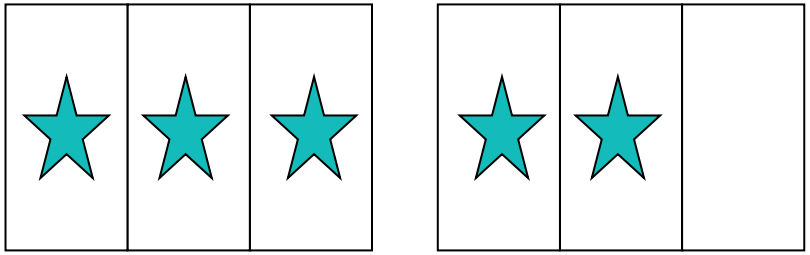
Incorporating Visual Models

- Using the fraction you previously created, create a number line to represent your fraction.
- Be sure to indicate the size of the fraction by showing the “jumps” between numbers.



Mixed Numbers

- Formally introduced in 4th grade
- "Experienced" in 3rd through visual models when exploring unit fractions
- The term "Improper Fraction" **does not** need to be used



$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{5}{3}$$

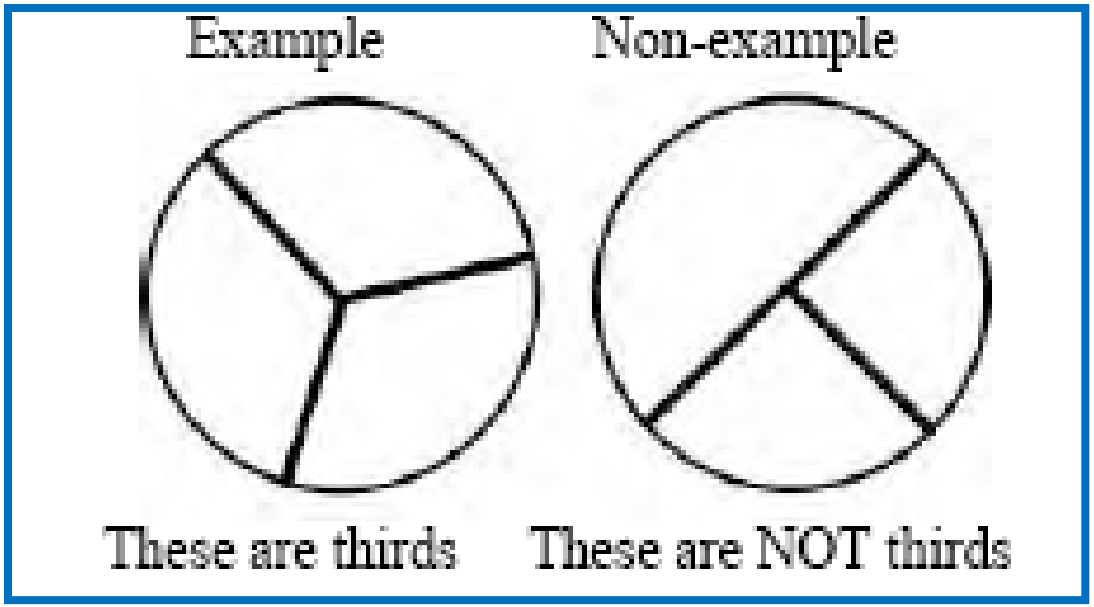
Comparing Fractions

In order to compare fractions they must have the same size whole.

Is ½ the same size for both these cakes?

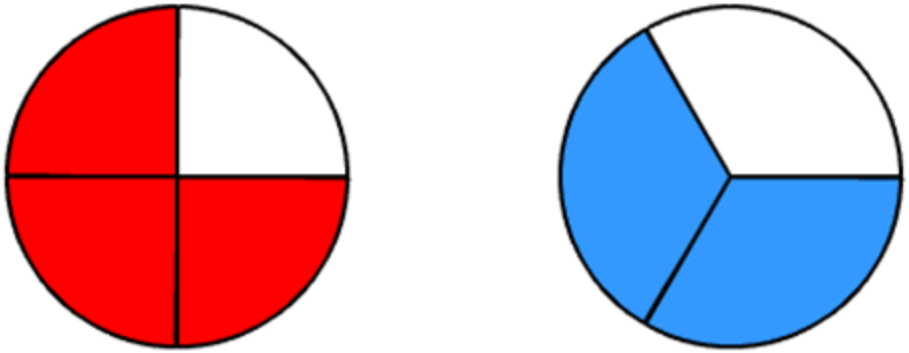
Comparing Fractions

In order to compare fractions the wholes must be divided equally.



Using Models to Compare

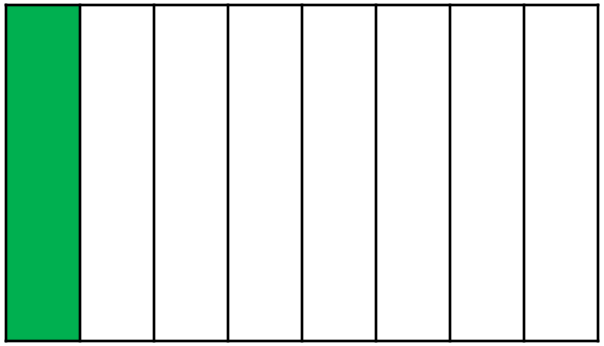
Student should use models to compare fractions.

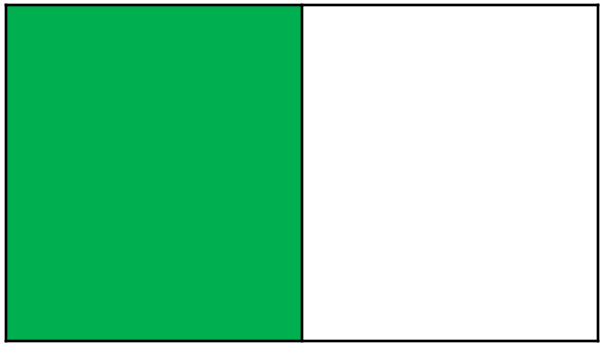


Which fraction shows more?

Using Reasoning to Compare

"1/8 is smaller than 1/2 because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces."





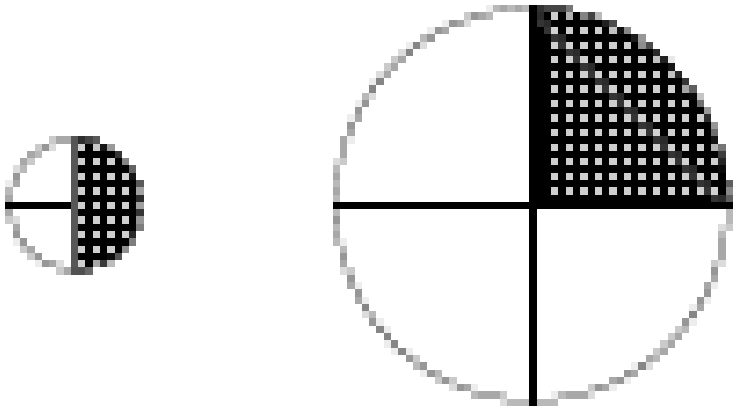
Which is bigger?

Which is bigger ¼ or ½?

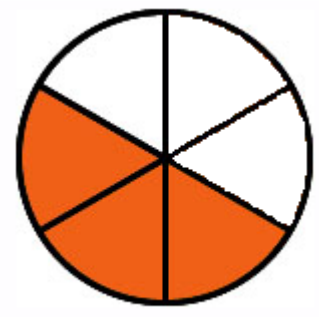
Explain your thinking to a tablemate.

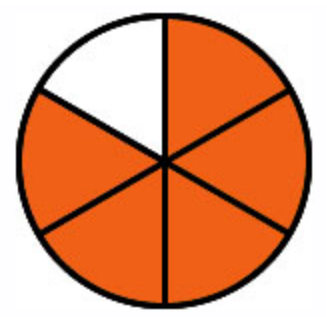
Equal Numerators

Now that you can see the size of the wholes, do you want to change your answer?



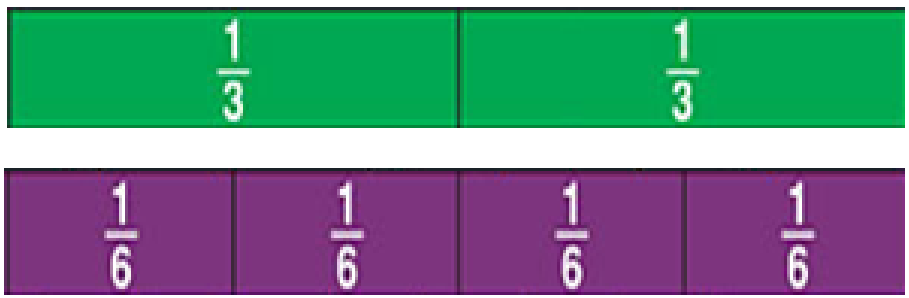
Equal Denominators

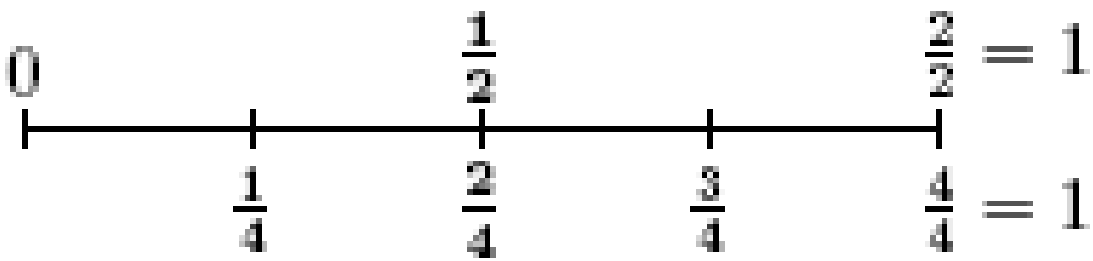




$$\frac{3}{6} < \frac{5}{6}$$

Equivalent Fractions Using Models





*3rd grade students find equivalent fractions **without** the use of an algorithm.

Let’s Explore Fractions

Use your fraction bars to find the answers to the questions below.

1. Find two fractions that are less then 1/3.
2. Find two fractions that are equal to 4/8.
3. Find four fractions that are greater than 2/3?

Be sure to use the fraction bars to prove your thinking.

Agree/Disagree Journal Prompt

Liz said that she would rather have 1/6 of a pizza instead of 1/4 of a pizza. Liz said that 1/6 is a bigger slice so that is why she wants 1/6 of a pizza. Marco disagrees with Liz and says that 1/4 of a pizza is bigger.

Who is correct and why? Make sure you justify your answer.

Review:
Important Fractional Concepts

- Fractional parts must be equal sized.
- The number of equal parts tell how many make a whole.
- As the number of equal pieces in the whole increases, the size of the fractional pieces decreases.
- The size of the fraction is relative to the whole.
- When a whole is cut into equal parts, the denominator represents the number of equal parts.
- The numerator of a fraction is the count of the number of equal parts.

Progression of Denominators

3rd Grade:	4th Grade:	5th Grade:
Fraction denominators 2, 3, 4, 6, & 8	Fraction denominators 2, 3, 4, 5, 6, 8, 10, 12, & 100	No restrictions on denominators

Critical Area- Fractions
(4th grade)

Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.

Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.


fractions
fraction
denominators
two
visual
using
number
whole
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e.g
understand
understanding
understandings
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multiplication
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Note
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multiply
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Recognize
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mixed
Compare
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parts
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Extend
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Examples
4.NF.2
4.NF.3
4.NF.5
4.NF.6
4.NF.7
8/8
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Developing
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expectations
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composing
roast
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relationship
benchmark
strategies
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eat
rewrite
length
answer
and/or
needed
built
numerators
compare
adding
developing
pounds
grade
subtract
many
common
differ
respective
equations
Solve
Add

Key Concepts about Fractions
(4th grade)

- Unit fractions
- Visual fraction models
- **Addition and Subtraction of fractions with like denominators**
- **Multiplication of fractions by a whole number**
  - Denominators 2, 3, 4, **5**, 6, 8, **10, 12, 100**
  - Use area models, number lines and set or collection models

Key Ideas
Fractions & Computation

- Must be taught conceptually and without algorithms or “tricks”
- Students use a variety of visual models to represent the concept
- Help students make SENSE of the problem

Models & Symbols

Fraction Number Sense:Representations

- Area/Region Models

- Linear or Measurement Models

- Set Models

- Symbols (with meaning)

$\frac{3}{4}$ $\frac{7}{8}$ $\frac{1}{2}$

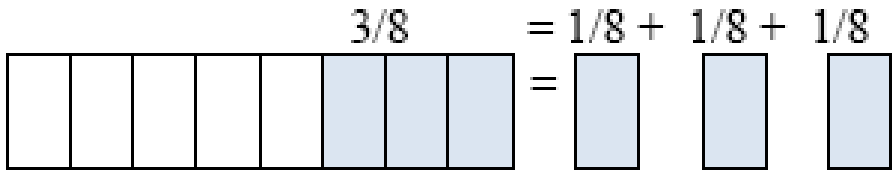
Addition & Subtraction

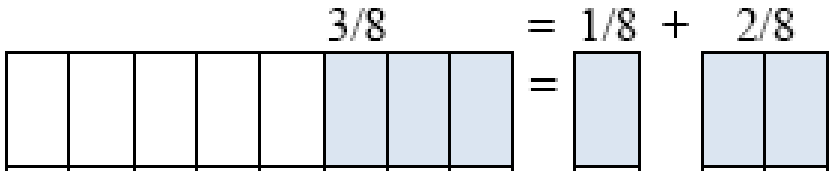
4th Grade

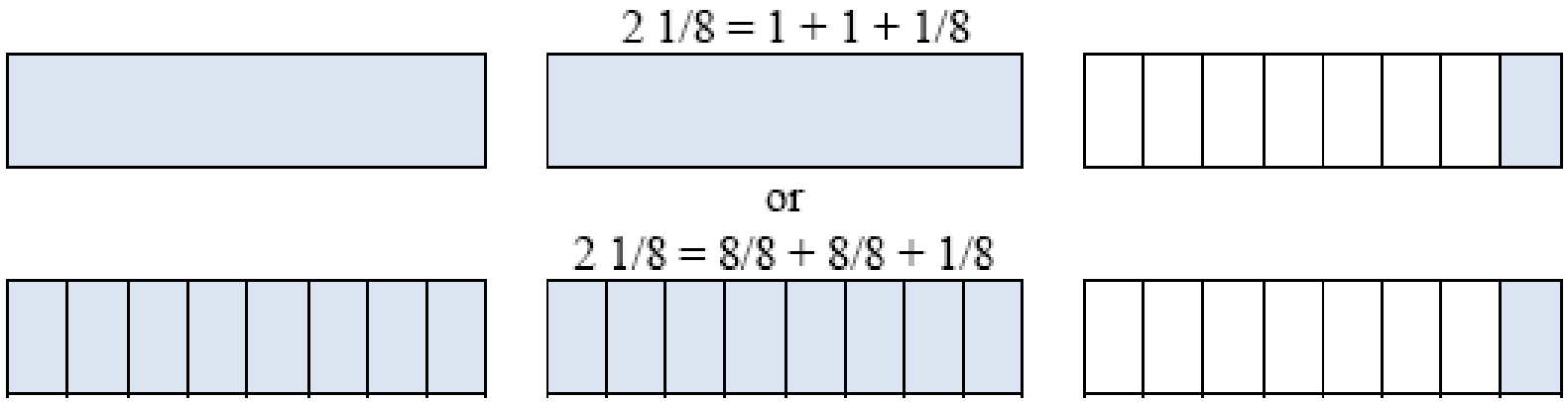
- Add/subtract fractions and mixed numbers with **LIKE** denominators
- Estimate using whole number benchmarks
- Build understanding with visual models *first*
  - Compose/decompose, fraction bars, number lines, rectangles
- Replace given mixed numbers with equivalent fractions
- Apply to real world contexts & problem solving
  - Estimation, benchmark fractions, judge reasonableness

Visual Fraction Models

Decomposing Fractions



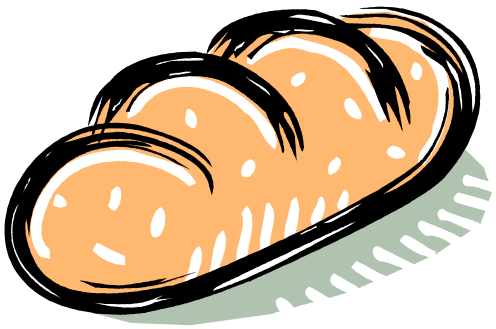




Adding & Subtracting Fractions

Solve the following problem by decomposing using a fraction model.

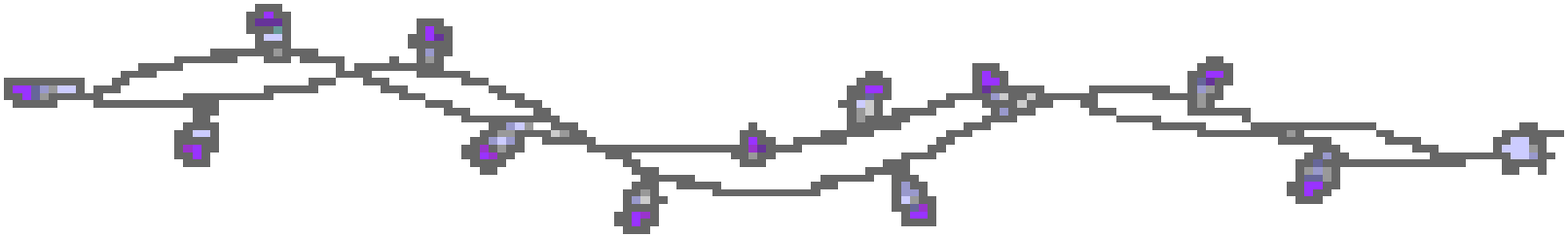
Julie baked 3 large loaves of bread. Ray and Sara each ate $\frac{3}{4}$ of a loaf. Eve ate $\frac{1}{4}$ of a loaf, and Amy ate $\frac{2}{4}$ of a loaf. How much bread is left? Explain your reasoning.



Adding & Subtracting Fractions

Solve the following problem by decomposing using a fraction model.

Rachel and Mike need $9\frac{7}{8}$ feet of outdoor lights to decorate their front porch. Rachel has $4\frac{2}{8}$ and Mike has $5\frac{6}{8}$. How many feet of lights do they have? Is it enough to decorate the front porch?



Decomposing Mixed Numbers

Students may need to decompose mixed numbers to help them add/subtract.

$$4\frac{2}{8} = \frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{1}{8} + \frac{1}{8}$$

$$5\frac{6}{8} = \frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{8}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 10$$

OR

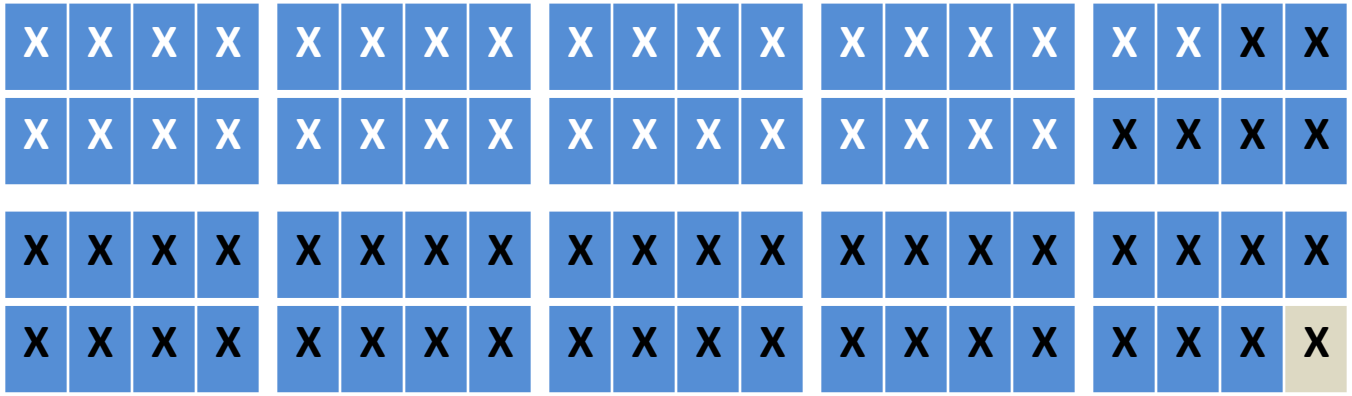
$$4\frac{2}{8} + 5\frac{6}{8} = \underbrace{4 + 5}_{} + \underbrace{\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}}_{}$$

$$= 9 + 1$$

$$= 10$$

Decomposing Mixed Numbers

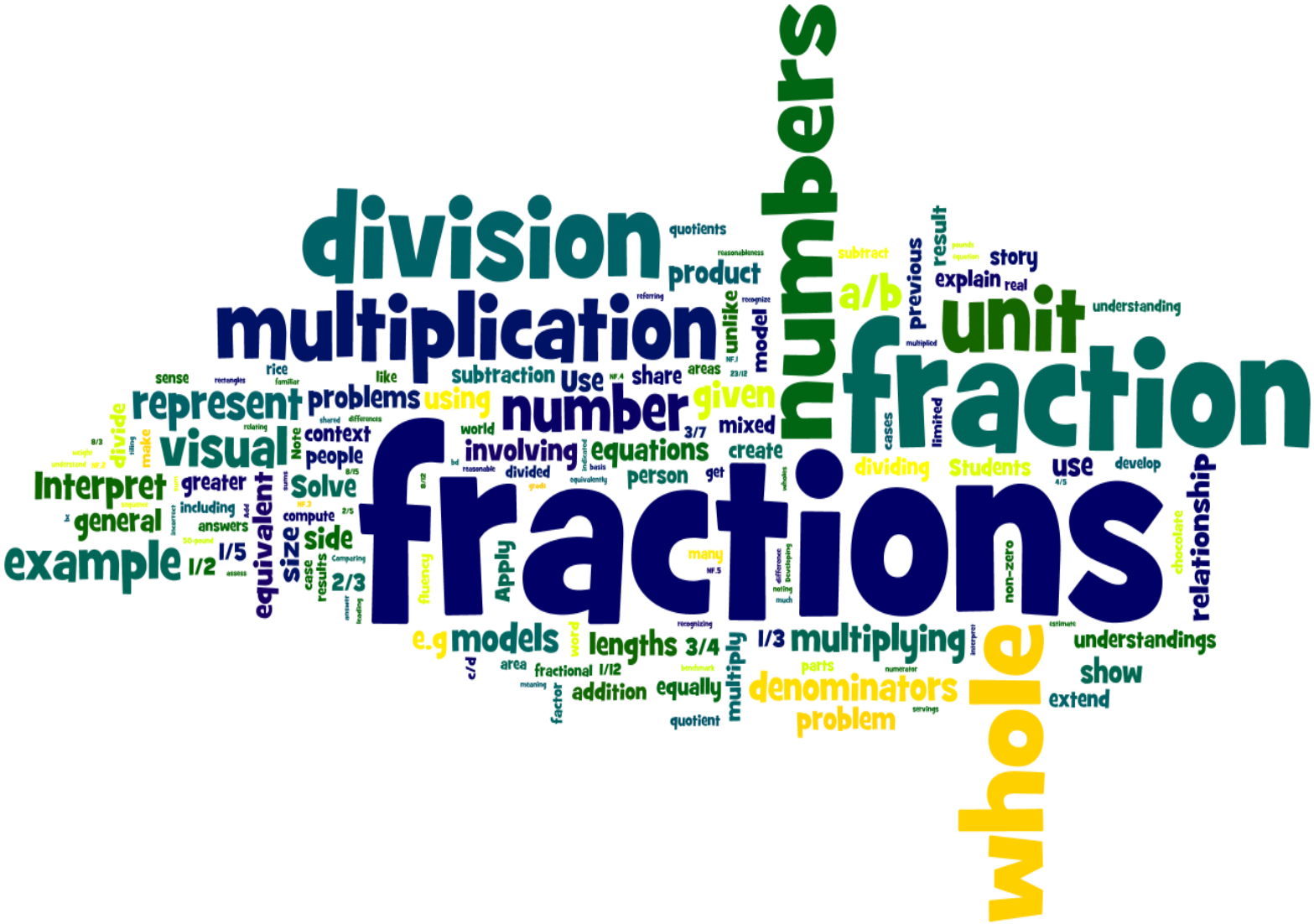
Rachel and Mike needed $9\frac{7}{8}$ feet of lights which is equivalent to $\frac{79}{8}$. The blue shaded boxes reflect the amount of lights needed. The white X's show Rachel's amount and the black X's show Mike's amount. Together they have 10 feet of outdoor lights. They have enough to cover the front porch. They actually have a little extra, $\frac{1}{8}$.



Critical Area: Fractions
(5th Grade)

Developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).

Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)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relating
differences
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understand
filling
sequence
incorrect
Add
50-pound
assess
Comparing
amount
heading
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grade
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bookkeeping
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recognizing
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equation
multiplied
model
unlike
world
reasonable
get
equally
multiply
2/7

Addition and Subtraction

5th Grade

- Add/subtract fractions and mixed numbers with UNLIKE denominators

- Estimate using benchmark fractions

- Build understanding with visual models *first*
  - Area models, number lines, fraction circles

- Replace given fractions with equivalent fractions
  - Common denominators

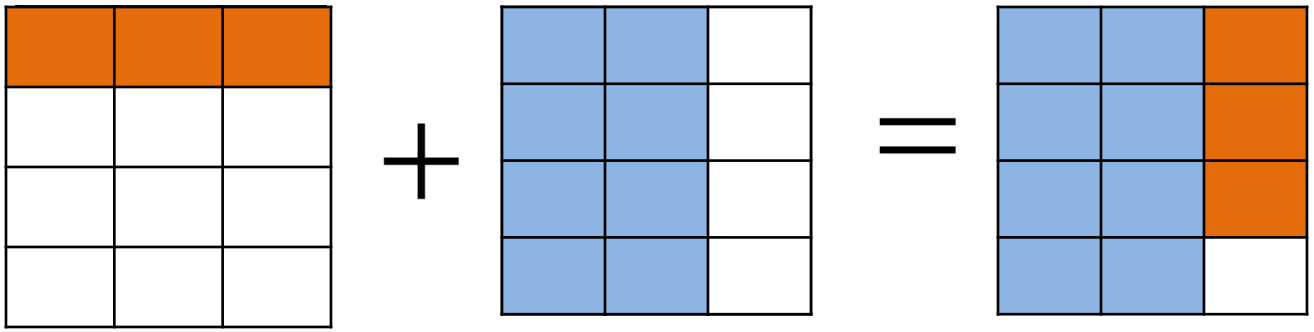
- Apply to real world contexts & problem solving
  - Estimation, benchmark fractions, judge reasonableness

Visual Fraction Models

$$\frac{1}{4} + \frac{2}{3}$$

Area Model

A model is needed that is divided into fourths & thirds.



$$\frac{1}{4} + \frac{2}{3} = \frac{11}{12}$$

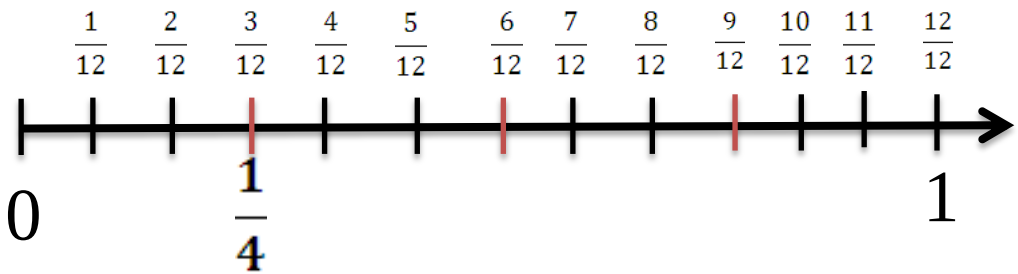
$$\frac{3}{12} + \frac{8}{12} = \frac{11}{12}$$

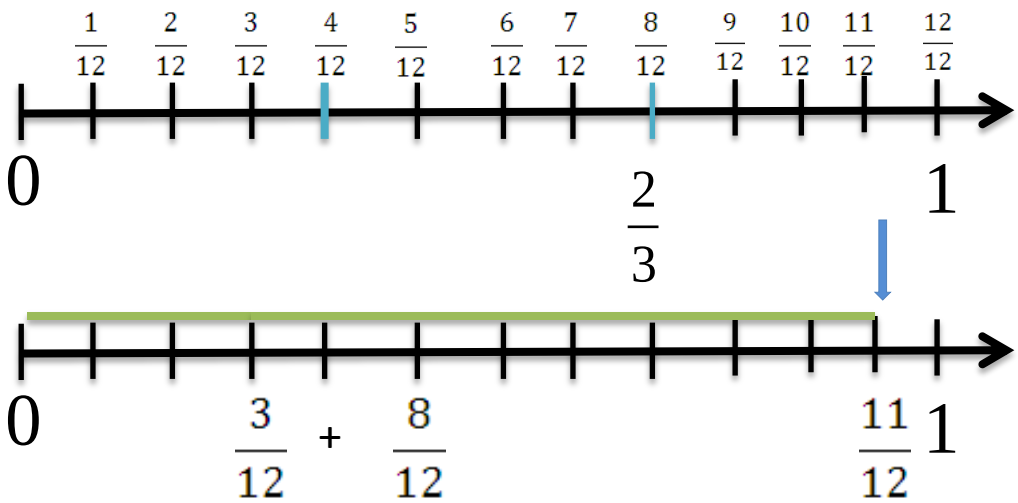
Visual Fraction Models

$$\frac{1}{4} + \frac{2}{3}$$

Number lines

Create a number line that encompasses fourths and thirds.





Real World Example

Carter is making his famous tropical punch. The recipe calls for $2\frac{3}{8}$ cups of pineapple juice. He already has $\frac{3}{4}$ of a cup of pineapple juice. How much more pineapple juice does he need to make his famous punch?

- ✓ Solve the problem above using an area model or number line.
- ✓ Compare your solutions to a person seated near you.

Common Denominators

- Again, students MUST use visual models first. This will help build an **understanding** about the efficiency of using common denominators.

- Apply **understanding** of *equivalent fractions* to find common denominators.

"It is **<u>not</u>** necessary to find a *least* common denominator to calculate sums of fractions. In fact, the effort of finding a *least* common denominator is a distraction from understanding adding fractions."

-DPI Unpacking Document

Fraction Operations

- The meaning of each operation with fractions is the same as the meaning for the operations with whole numbers.
- Answers don't *always* get bigger with multiplication and smaller with division!
- Knowing THAT an algorithm works is not the same thing as knowing WHY an algorithm works.
- Allow students to contextualize & decontextualize.

Fractions & Multiplication

What makes fraction multiplication so challenging for students?

What “tricks” have been helpful?

The Content
Fractions & Multiplication

4th Grade

- Multiply a fraction by a whole number
- Introduced as an extension of repeated addition

5th Grade

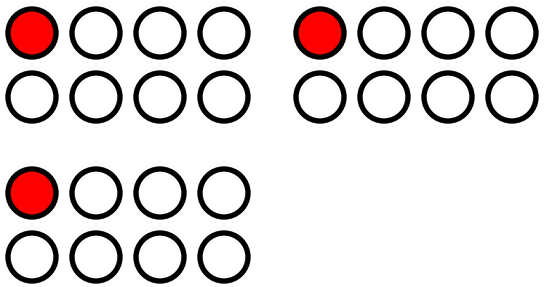
- Multiply a fraction by a whole number
- Multiply a fraction by a fraction
- Create story contexts (real world applications)
- Includes mixed numbers
- Understand how numbers change when we multiply fractions
- Relate the operations back to what they know about whole numbers

Visual Models Used to Represent Multiplication with Fractions

Which visual models have you used with students to multiply with fractions?

Multiplying a Fraction by a Whole Number

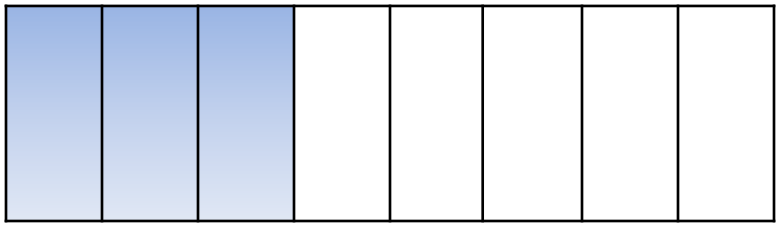
$$\frac{1}{8} \times 3$$



$$\frac{1}{8} \times 3 = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

We have 3 groups of $\frac{1}{8}$ which is $\frac{3}{8}$

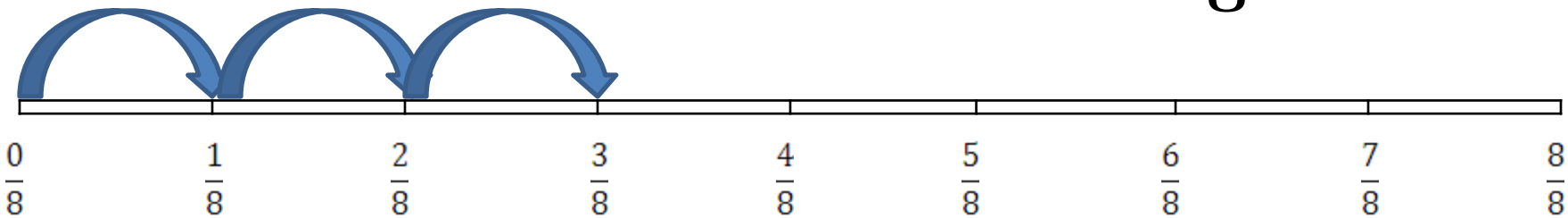
$\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$



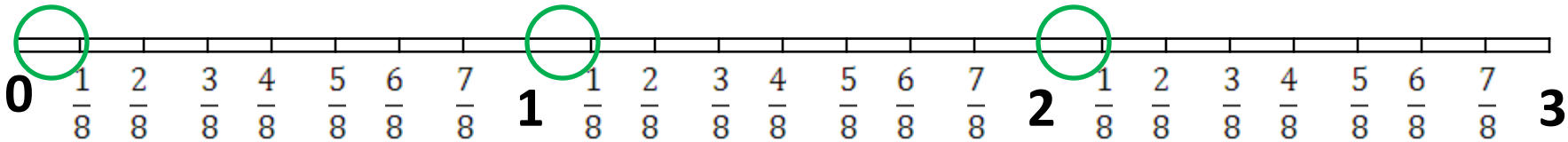
Try modeling this using a number line.

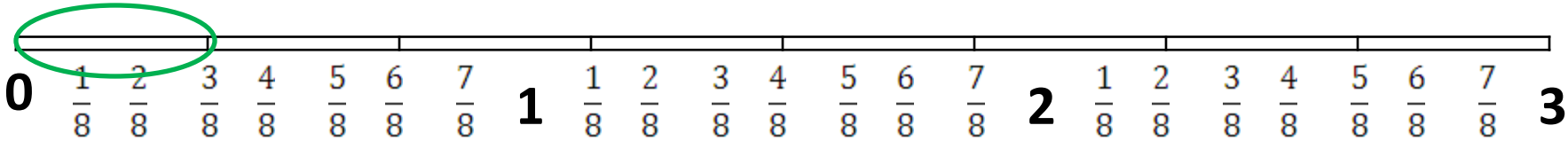
Multiplying a Fraction by a Whole Number

$$\frac{1}{8} \times 3$$



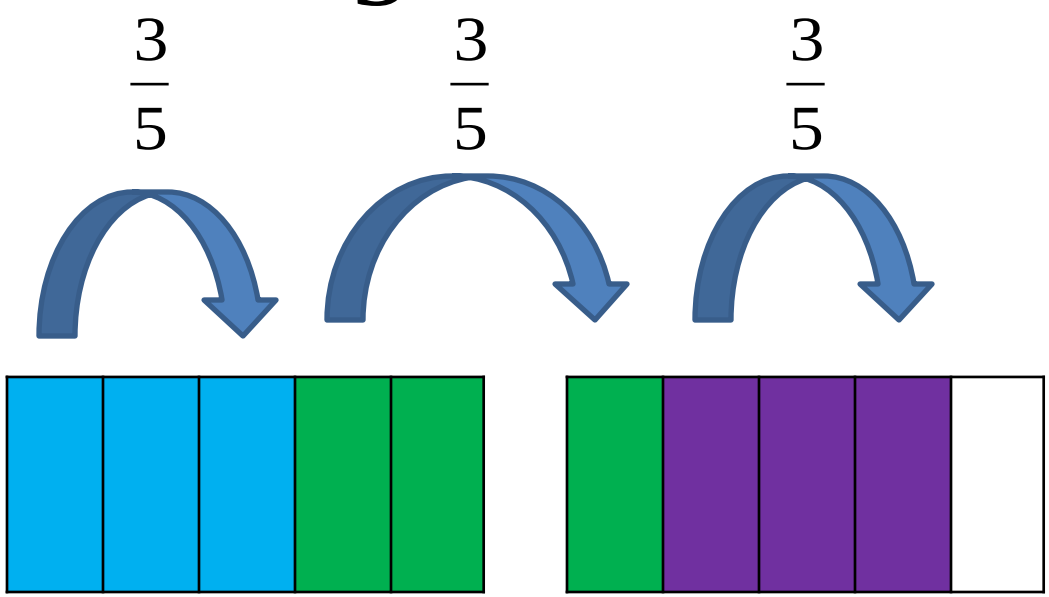
$$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$





Multiplying a Fraction by a Whole Number

$$\frac{3}{5} \times 3 = 1\frac{4}{5}$$



Try modeling this using another strategy we've looked at (number line, decomposing, visual models, etc).

Multiplying a Fraction by a Whole Number in Context

Create a story context for the expression below:

$$\frac{3}{4} \times 6$$

Multiplying a Fraction by a Whole Number in Context

$$\frac{3}{4} \times 6$$

***Possible story context*:**

Gavin poured $\frac{3}{4}$ of a gallon of water in each of the canteens he packed for a hiking trip. If he packed 6 canteens, how much water did he pack for his trip?

Create a visual model to help make sense of this problem.

http://www.fotosearch.com/dg-vinyl-clip-art/camping-hunting-fishing/SUE107/3/

Multiplying a Fraction by a Fraction

Work with your table group to **CONTEXTUALIZE** this:

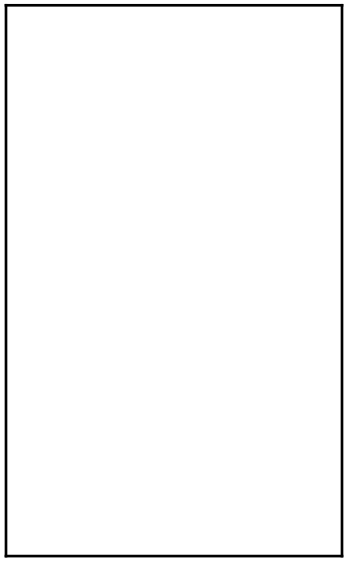
$$\frac{1}{3} \times \frac{1}{4}$$

Multiplying a Fraction by a Fraction

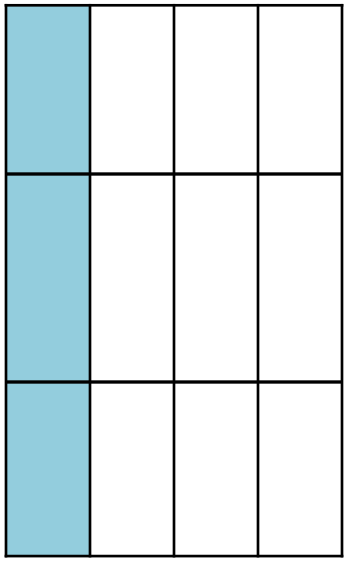
$$\frac{1}{3} \times \frac{1}{4}$$

THINK: What is $\frac{1}{3}$ of $\frac{1}{4}$?

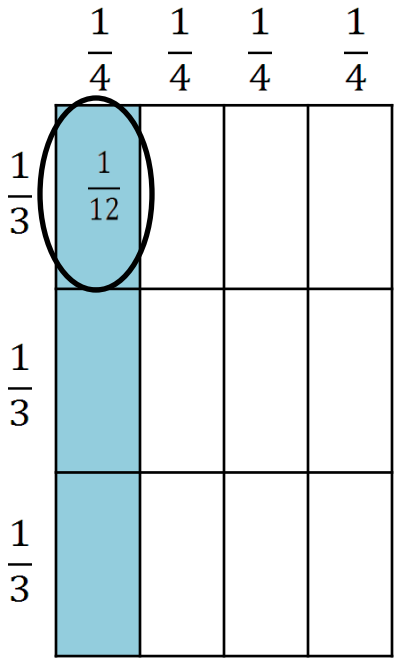
Suppose you have a cake and you want to find one third of a fourth of the cake.



You divide the cake in fourths and then you divide each $\frac{1}{4}$ section into thirds.



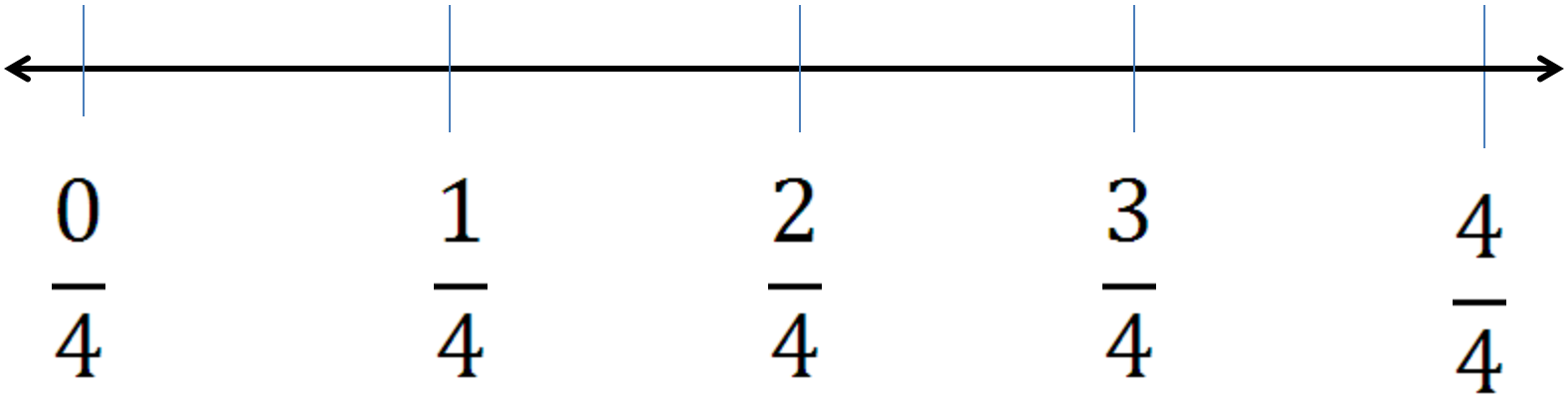
You can see that $\frac{1}{3}$ of one fourth is $\frac{1}{12}$.



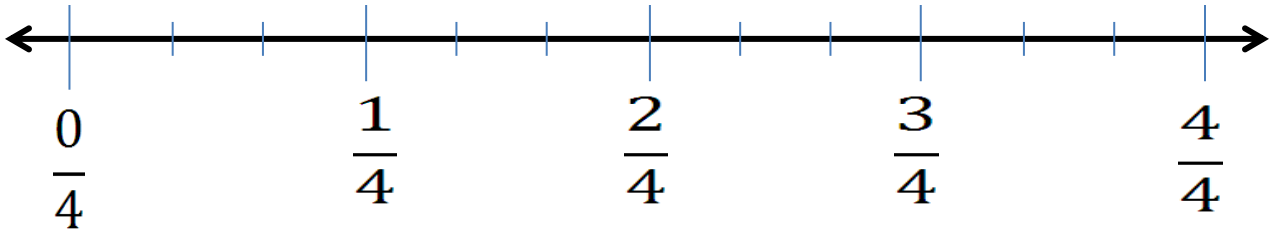
Multiplying a Fraction by a Fraction

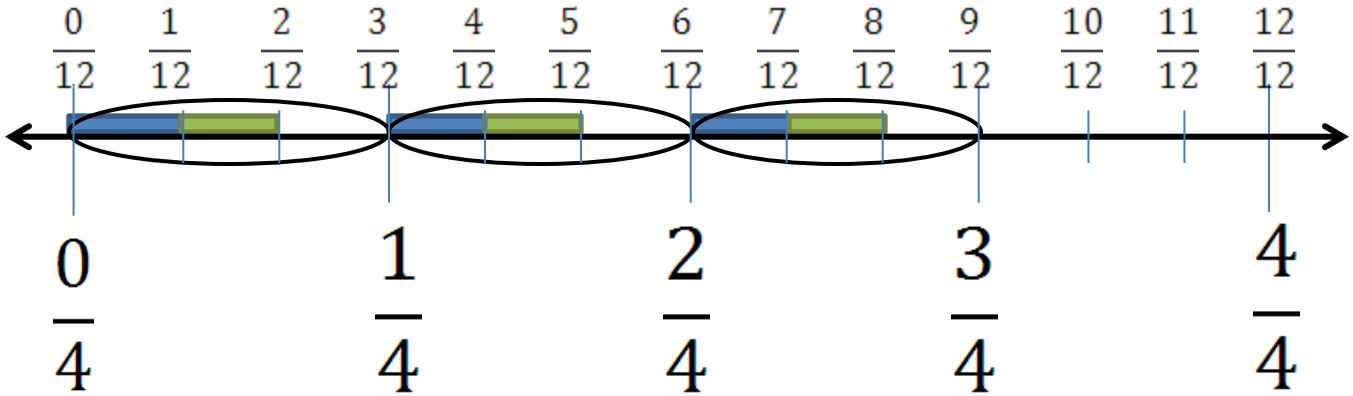
Brenden had $\frac{3}{4}$ of a board. He decided to use $\frac{2}{3}$ of the board to make a shelf for his book case. How much of the board did Brenden use to make a shelf?

How could you use the number line below to solve this problem?



Multiplying a Fraction by a Fraction



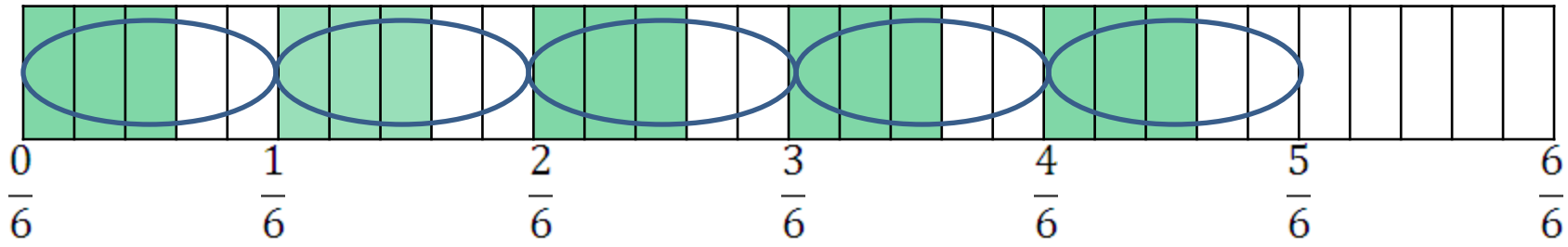


So $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$ or $\frac{2}{3}$ ***of*** $\frac{3}{4}$ is $\frac{6}{12} = \frac{1}{2}$

Multiplying a Fraction by a Fraction

Which equation is modeled below?

Can you create a context for this equation?



Multiplying a Fraction by a Fraction

Using the models with mixed numbers.

- How can you model the following problem?
- Find one solution and if you have time, use a different model to find the answer.

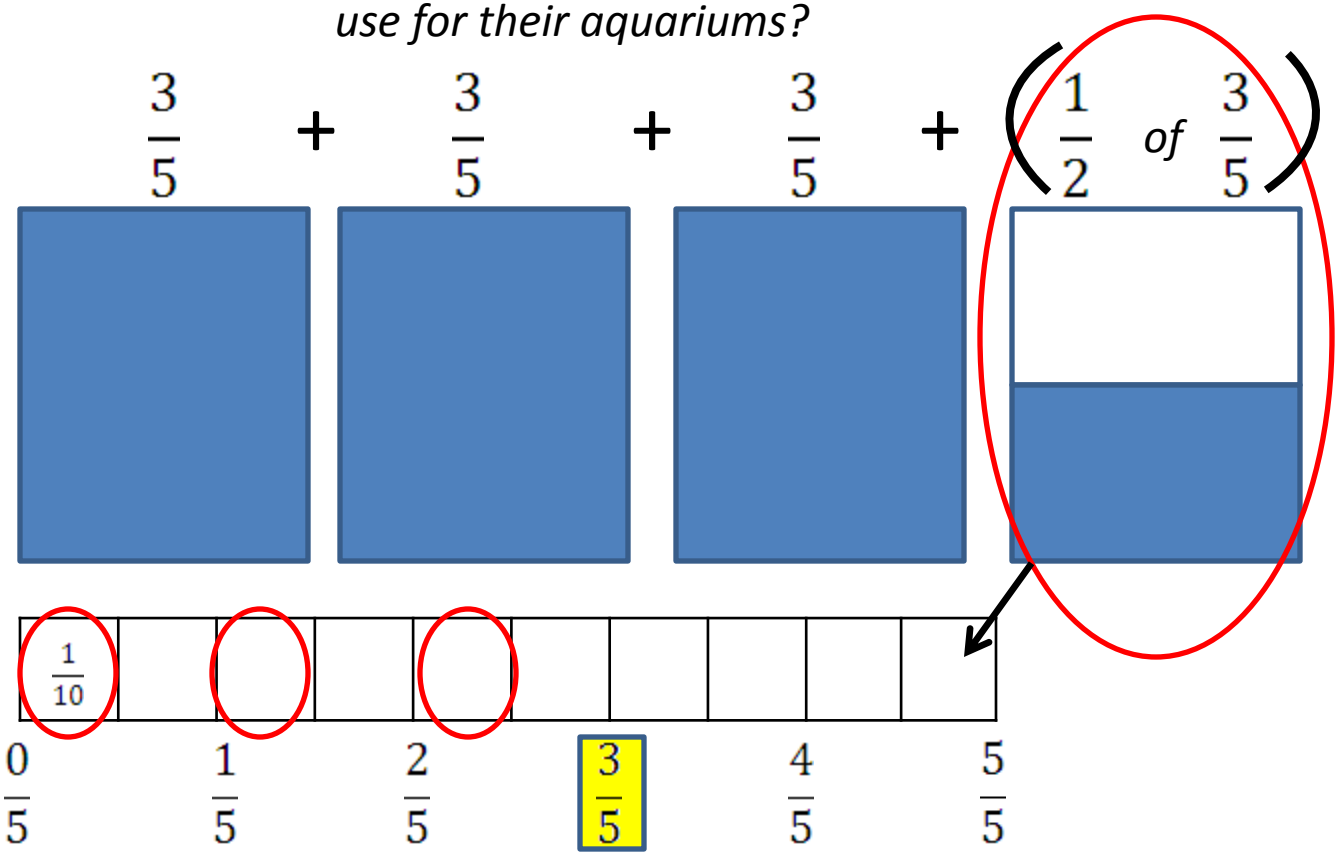
Mr. Johnson's class used $3\frac{1}{2}$ *bags of gravel for their aquariums. Each bag of gravel weighs* $\frac{3}{5}$ *of a pound. How many pounds of gravel did Mr. Johnson's class use for their aquariums?*

Multiplying a Fraction by a Fraction

Using the models with mixed numbers.

Mr. Johnson's class used $3\frac{1}{2}$ *bags of gravel for their aquariums. Each bag of gravel weighs* $\frac{3}{5}$ *of a pound. How many pounds of gravel did Mr. Johnson's class use for their aquariums?*

$$\frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \left(\frac{1}{2} \text{ of } \frac{3}{5}\right)$$



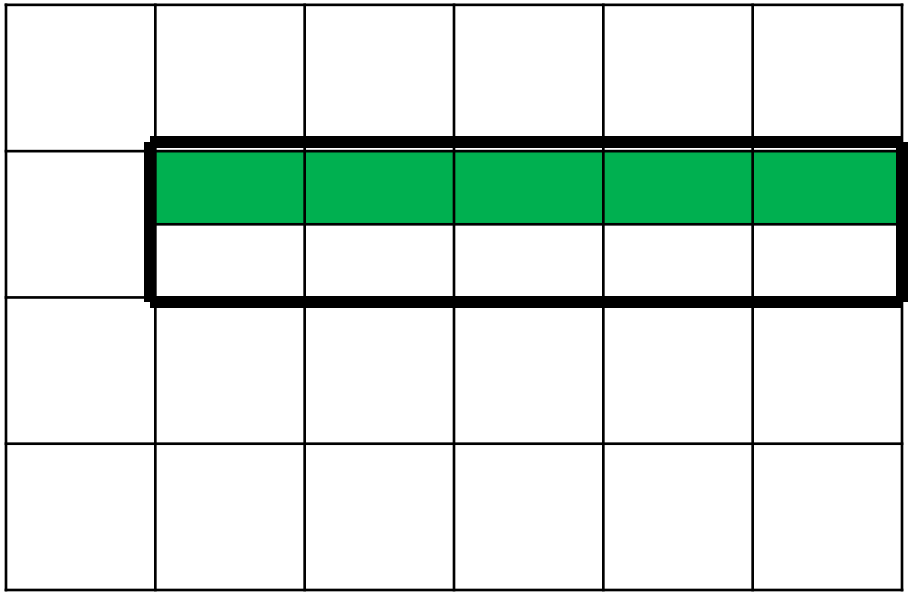
$$\begin{array}{r} \boxed{\frac{3}{5}} \\ \frac{3}{5} \\ \frac{3}{5} \\ + \quad \frac{3}{10} \\ \hline \end{array}$$

$\frac{21}{10}$ *or* $2\frac{1}{10}$ *lbs*

Area of Rectangle with Fractional Sides

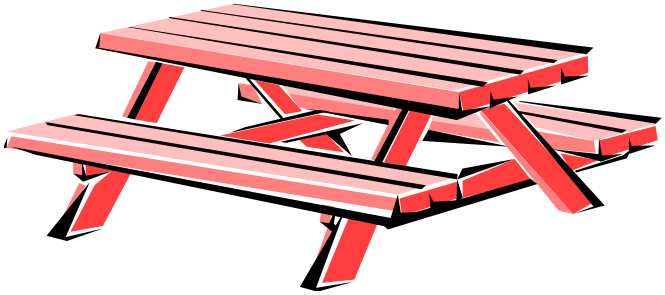
What equation is modeled?

Can you create a context for this equation?



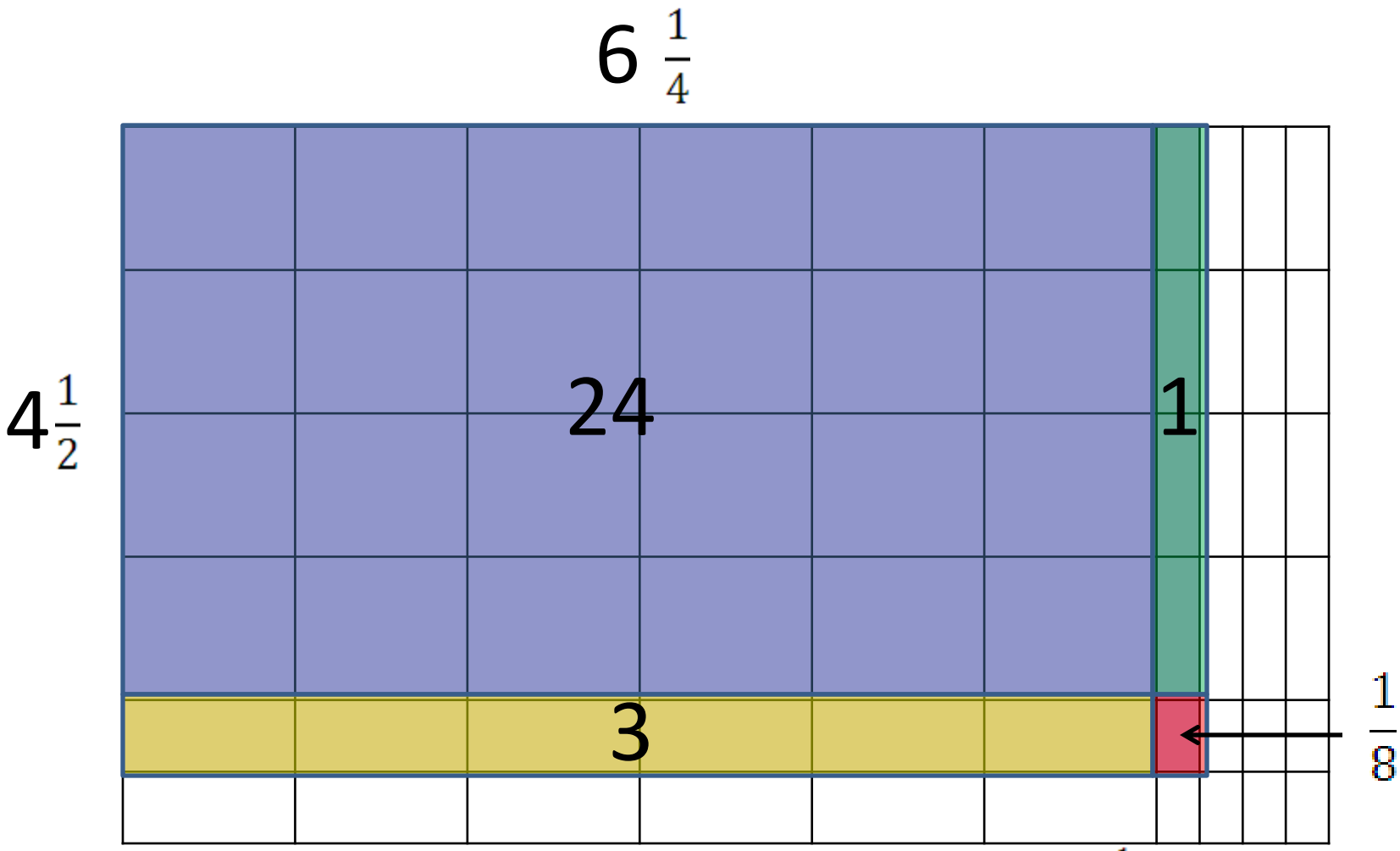
Multiplying Mixed Numbers

The picnic tables at Bond Park are 4 ½ feet wide and 6 ¼ feet long. Ms. Long will bring a table cloth with an area of 26 square feet when she has a picnic at Bond Park this weekend. Will this table cloth be large enough to cover a picnic table? Why or why not?



Multiplying Mixed Numbers

using the area model



$24 + 3 + 1 + \frac{1}{8} = 28\frac{1}{8}$

Multiplying Mixed Numbers

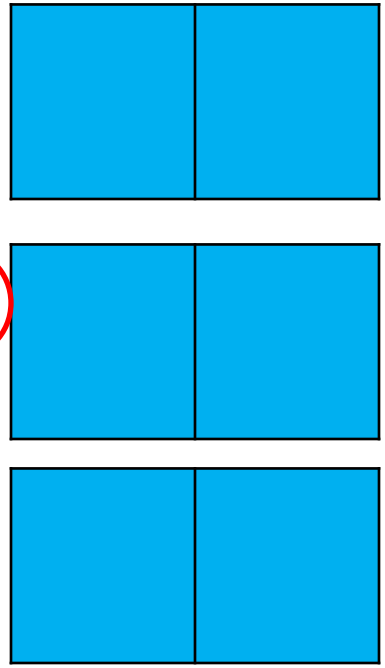
by decomposing

$6\frac{1}{4} \times 4\frac{1}{2}$

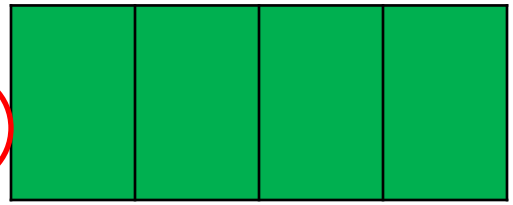
$6 \quad \frac{1}{4}$

$6 \times 4 = 24$

$6 \times \frac{1}{2} = 3$



$\frac{1}{4} \times 4 = 1$



$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

$6\frac{1}{4} \times 4\frac{1}{2} = 28\frac{1}{8}$

Multiplying Mixed Numbers

practice area model & decomposing

Solve the following word problem using the area model and by decomposing.

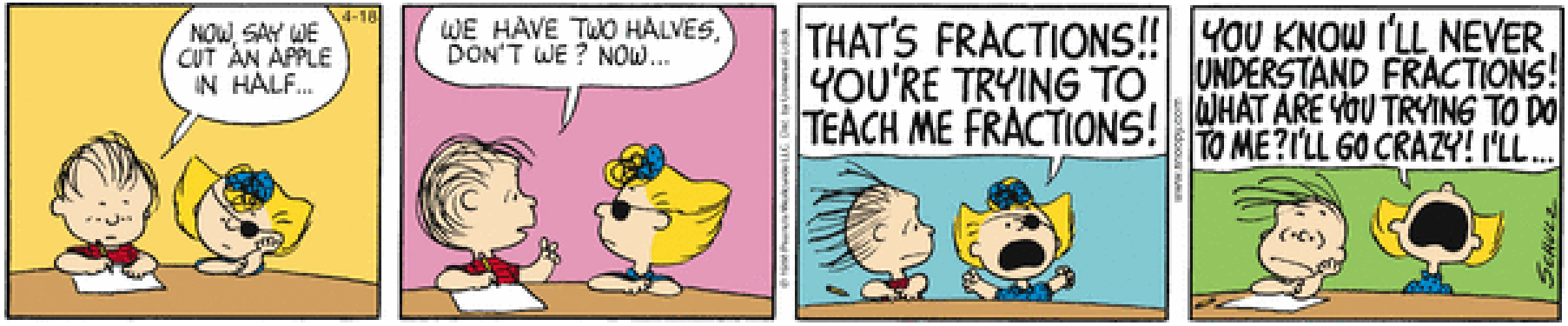
Joan is making a poster to take with her to her favorite band's concert. The poster she has to decorate is $3\frac{2}{3}$ *feet by* $2\frac{2}{5}$ *feet. What is the area of the poster Joan has to decorate?*

Fractions & Division

What makes division with fractions so challenging for students?

What “tricks” have been helpful?

Division & Fractions



- Interpret fractions as division

- Divide unit fractions by whole numbers or whole numbers by unit fractions

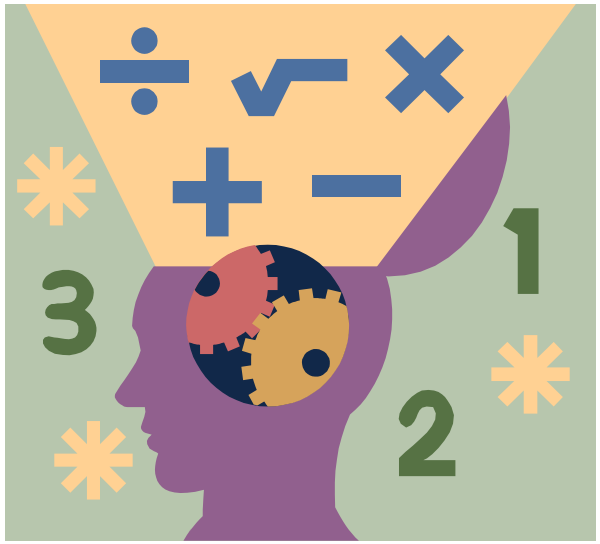
http://www.gocomics.com/peanuts/2013/04/18 (graphic)

When I divide, my quotient is _____ than the dividend.

A. Smaller

B. Bigger

C. Not enough information

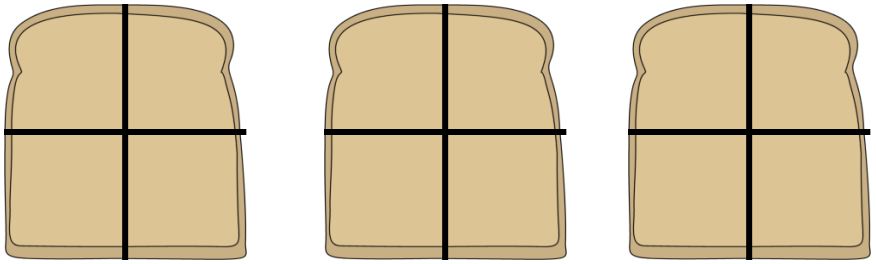


Interpret a Fraction as Division

Joe had 3 sandwiches in his picnic bag. There were 4 people at the picnic. Each person will receive what part of a fraction?

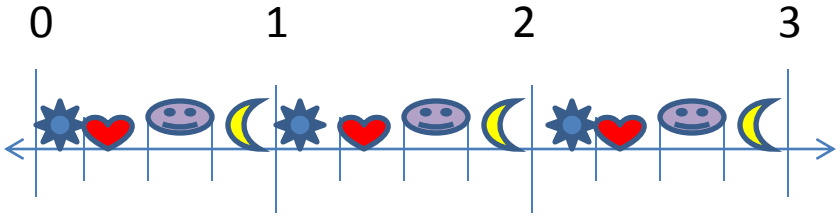
$$\frac{3}{4} = 3 \div 4$$

Visual Model



$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ of a sandwich

Number Line



Break section into 4 parts, then share the parts among the 4 people. Each receives $\frac{3}{4}$ of a sandwich.

Dividing a Whole Number by a Unit Fraction

The guppies from your ecosystem unit are living in a five gallon aquarium. When it is time to change the water, you use a one quart graduated cylinder ($\frac{1}{4}$ gallon) to refill the tank. How many times will you need to fill the graduated cylinder to fill the 5 gallon tank?

Solve using an equation and a visual fraction model.

Picture Proof

$5 \div \frac{1}{4}$

1 aquarium = 5 gallons

1 aquarium = 5 gallons

5 gallons ÷ $\frac{1}{4}$ gallon graduated cylinders = 20 graduated cylinders to fill the aquarium

Visual Fraction Model

1 gallon
1 gallon
1 gallon
1 gallon
1 gallon

The aquarium has 5 gallons in it

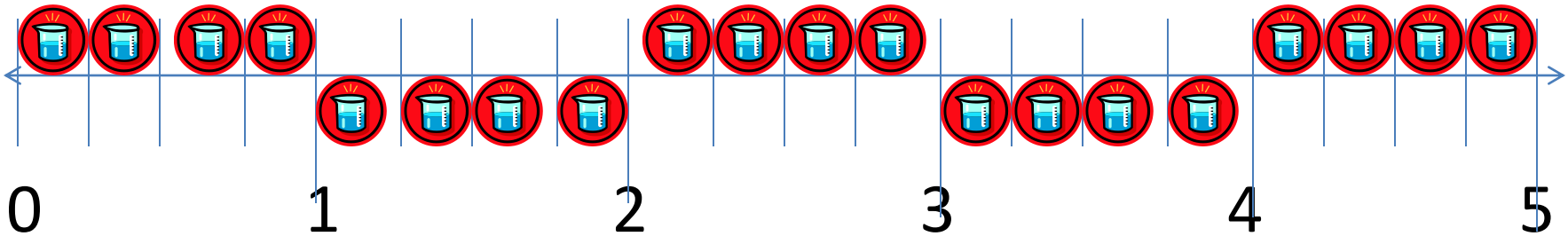
1 gc	1gc	1gc	1gc

Each gallon will take 4 graduated cylinders to fill.

4 x 5 = 20 cylinders

$5 \div \frac{1}{4}$

Number Line



- Count to five on the number line.
- Break each section into 4 equal parts.
- $5 \div \frac{1}{4} = 20$ because 5 groups of 4 is 20

Put Your Heads Together!

- With a partner or your table, contextualize a problem that will result in a whole number divided by a unit fraction (bonus points if it relates to a science unit!).
- Once the problem is written, solve it in as many ways as you can – either individually or assign each person a method.
- Be ready to share out which skills you have used in this process.

Divide Unit Fraction by Whole Number

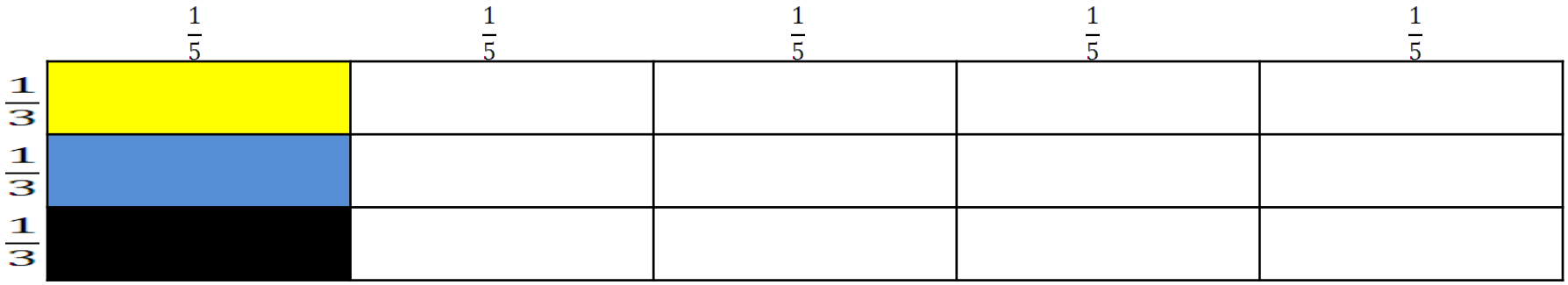
Mr. Ramirez had $\frac{1}{5}$ of a pound of strawberries left after he baked a pie. He wants to split the strawberries among the three people in his family for dessert tonight. How much of a pound will each person receive?

How many different ways can you solve this problem?

Using Visual Fraction Models

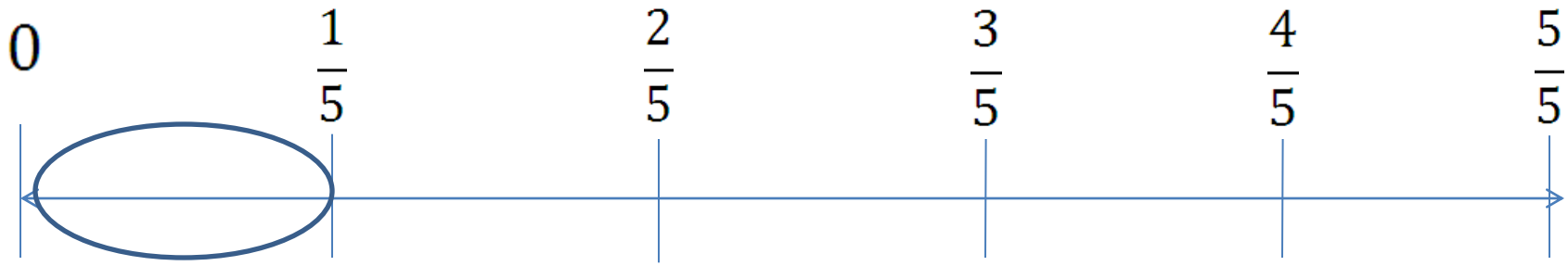
What does Mr. Ramirez start with in this problem?

What does he do with his strawberries?

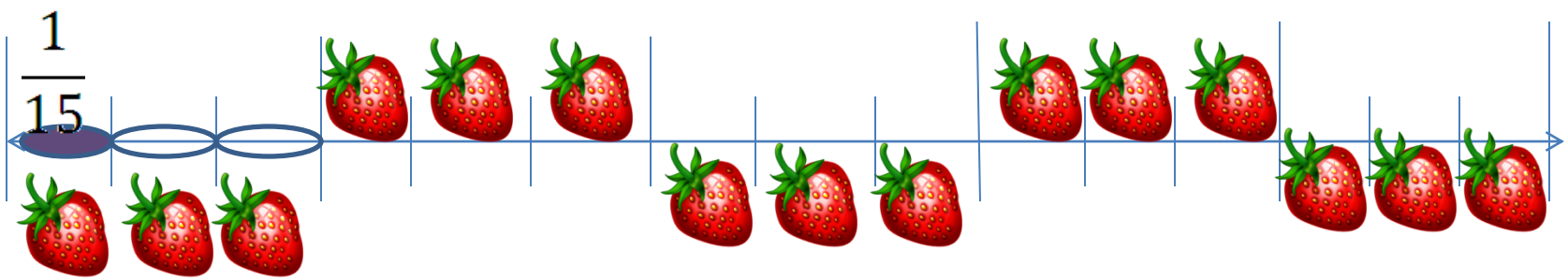


He splits the $\frac{1}{5}$ into three equal parts, or finds $\frac{1}{3}$ of $\frac{1}{5}$, giving each person $\frac{1}{15}$ of a pound of strawberries.

Using a Number Line



Start by identifying $\frac{1}{5}$ on the number line. That is the amount of strawberries Mr. Ramirez has to split with his family.



Within that one fifth, identify three equal parts.
Each person in the family will receive one of the parts. That part is $\frac{1}{15}$ of the entire whole. Each person will receive $\frac{1}{15}$ of a pound of strawberries for dessert.