Using Classroom Talk to Support the Standards for Mathematical Practice

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Agenda

- Process for Statistical Investigation PCAI Model (55 min)
  - Estimating Length of String
- Promoting Discourse and the SMP (40 min)
- Short Break (10 min)
- Exploring the Olympics (40 min)
- Questions/comments (5 min)
How well can a group of teachers estimate the length of string?

Adapted from Effect of Prior Information on String Length Estimates (Franklin & Mulekar, 2004)
How well can we estimate lengths?

- On your own …
  - Write your estimate for the length of string 1 on a sticky note. Do not change your estimate.
  - Write your estimate for the length of string 2 on a different color sticky note. Do not change your estimate.

- With a partner …
  - Discuss ways to organize the data so you can compare the two data sets. How are the two data sets similar? How are the two data sets different?
A histogram is an estimate of the density of data. The height of the bar in each bin is proportional to the number of data points that have values in that bin. A histogram is determined not only by the bin width, but also by the choice of an anchor (or origin).

The interquartile range (IQR) is the width of the box in the box-and-whisker plot. The IQR can be used as a measure of how spread-out the values are.

An outlier is any value that lies below $Q_1 - 1.5 \times IQR$ or above $Q_3 + 1.5 \times IQR$ and is viewed as being too far from the central values to be reasonable.

How well can we estimate lengths?

☐ As a whole group …
  - Represent the class data using a histogram so you can compare teachers’ estimates of string length.
  - Represent the class data using a box plot so you can compare teachers’ estimates of string length.

☐ On your own …
  - Write some statements about your class data. Note any patterns you see.

☐ In a small group …
  - Share your observations with your partner and another pair. Answer the questions, and be prepared to share your arguments with the whole group.
A Larger Sample
Questions to Guide Small Group Discussions

- What patterns did you notice about the data?
- What is the overall shape of the distribution?
- Where is the center of the distribution?
- How would you describe the spread of the distribution?
- Compare the typical estimations for string 1 and string 2. How are they similar? How are they different?
- If a new teacher joined our group today, what length would you predict the teacher to estimate for string 1? Why?
Process of Statistical Investigation

PCAI Model
- Pose a question
- Collect data
- Analyze data
- Interpret results

(Graham, 1987)

The four components of the PCAI model may emerge linearly, or may include revisiting and making connections among the components.
Posing a Question

Select a question that

- is motivated by describing summarizing, comparing, and generalizing data within a context
- is measurable
- anticipates variability
Posing a Question

Statistical questions

- focus on a census of the classroom in elementary school (GAISE Report, 2007)

- often require cycles of iteration with data collection to get the question “right”
  - “How tall am I?”
  - “How tall are the people in my class?”
Collecting Data

Determine

- the *population*
  - full set of people or things that the study is designed to investigate

- methods of collecting data
  - *sample*, a subset of the entire population
  - *census*, the entire population

- if a sample will be collected, decide the size and how many

- if class data will be pooled consider representativeness and bias
  - *random samples* have characteristics that are representative of the population
Collecting Data

How do students see data?

- Data as a **pointer**
- Data as a **case**
- Data as a **classifier**
- Data as an **aggregate**

(Konold & Higgins, 2003)
Analyzing Data

Describe and summarize data

- using relevant summary statistics, such as the mean, median, mode and
- using tables, diagrams, graphs, or other representations

Describe variation

- measurement variability
- natural variability
- induced variability
- sampling variability
Interpreting Data

- Relate analysis to original question and context
- Make decisions about the question posed within the context of the problem based on data collection and analysis
Concept Map – PCAI

Process of Statistical Investigation

- Pose the question(s)
- Collect the data
- Interpret the results
- Analyze the distribution(s)
String Activity

- What question did we investigate?
- How did we collect data?
- How did we organize and analyze the data?
- What interpretations did we make about the data?
Discussion

- How does the process of statistical investigation, using the PCAI model, have the potential to support students’ in engaging in the SMPs?
- Which grade level standards have the potential to be addressed by the Estimating String Length activity?
- How might you modify the activity for your specific students?
Promoting Discourse in the Mathematics Classroom
Making the Case for Meaningful Discourse: Standards for Mathematical Practice

- Standard 1: Explain the meaning and structure of a problem and restate it in their words
- Standard 3: Habitually ask “why”
  - Question and problem-pose
  - Develop questioning strategies ...
  - Justify their conclusions, communicate them to others and respond to the arguments of others
- Standard 6: Communicate their understanding of mathematics to others
  - Use clear definitions and state the meaning of the symbols they choose
- Standard 7: ...describe a pattern orally...
  - Apply and discuss properties
Standards for Mathematical Practice

#3 Construct viable arguments and critique the reasoning of others

- Justify solutions and approaches
- Listen to the reasoning of others
- Compare arguments
- Decide if the arguments of others make sense
- Ask clarifying and probing questions
“Teachers need to develop a range of ways of interacting with and engaging students as they work on tasks and share their thinking with other students. This includes having a repertoire of specific kinds of questions that can push students’ thinking toward core mathematical ideas as well as methods for holding students accountable to rigorous, discipline-based norms for communicating their thinking and reasoning.”

(Smith and Stein, 2011)
Teacher Questioning

Teacher questioning has been identified as a critical part of teachers’ work. The act of asking a good question is cognitively demanding, it requires considerable pedagogical content knowledge and it necessitates that teachers know their learners well.

(Boaler & Brodie, 2004, p.773)
Levels of Analysis and Sense making

Phase 1: Making Thinking Explicit
(Explaining Reasoning)

Phase 2: Analyzing Each Other's Solution
(Analyzing Low Level to More Sophisticated Reasoning)

Phase 3: Developing New Mathematical Insights
(Abstract Mathematical Concepts)

From *Whole Class Mathematics Discussions*, Lamberg, Pearson 2012
Phase I: Make Thinking Explicit

- Turn and talk: What does this mean in the classroom?
Phase 1: Make Thinking Explicit

Active Listening is an important part of understanding someone else’s solution.

☐ Student should ask questions, if they are unclear about an idea presented.

☐ Teacher should monitor understanding of group by asking questions of explanation presented.

From Whole Class Mathematics Discussions, Lamberg, Pearson 2012
Questions to Clarify Understanding of Explanation

☐ Does everyone understand _____’s solution?
☐ Who can explain what ______ is thinking?
☐ Who would you like to help you explain?
☐ Can someone explain what _____ is thinking?
☐ Anyone confused about what he/she is saying?

From Whole Class Mathematics Discussions, Lamberg, Pearson 2012
Phase 2: Analyzing each other’s solutions to make mathematical connections

When students are expected to analyze each other’s solutions, they have to pay attention to what the students are saying. In addition, they need to think if the student’s explanation makes sense.

This requires students to make mathematical connections between ideas presented.

From *Whole Class Mathematics Discussions*, Lamberg, Pearson 2012
PHASE 2: ANALYZING EACH OTHER'S SOLUTION

Teacher questions to promote analysis and reflection of each other’s solutions:

- What do you see that is the same about these solutions?
- What do you see that is different about these solutions?
- How does this relate to ___?
- Ask students to think about how these strategies relate to the mathematical concept being discussed

From Whole Class Mathematics Discussions, Lamberg, Pearson 2012
Teacher Questions to Promote Mathematical Insights

- Ask students to summarize key idea.
- Ask questions: Will the rule work all the time? (Making generalizations)
- Introduce vocabulary or mathematical ideas within the context of conversation
- Ask students to solve a related problem that extends the insights they had gained from the discussion.
- Ask “What if” questions.

From Whole Class Mathematics Discussions, Lamberg, Pearson 2012
In the Classroom

- Video: Statistical Analysis to Rank Baseball Players
- www.teachingchannel.org

- What evidence of understanding is seen in the student discussions?
Short Break
5 Practices for Orchestrating Productive Mathematics discussions

- Determine the goal—what you want the students to learn
- Choose a rich task to help students attain the goal

The Five Practices:
1. Anticipating
2. Monitoring
3. Selecting
4. Sequencing
5. Connecting

Adapted from 5 Practices for Orchestrating Productive Mathematics Discussions, Smith and Stein, NCTM, 2011
Anticipating

- Anticipating Students’ Responses
  - What strategies are students likely to use to approach or solve a challenging, high-level mathematical task
  - How to respond to the work that students are likely to produce
  - Which strategies from student work will be most useful in addressing the mathematical goals
Monitoring

- Monitoring is the process of paying attention to what and how students are thinking during the lesson
- Students working in pairs or groups
- Listening to and making note of what students are discussing and the strategies they are using
- Asking questions of the students that will help them stay on track or help them think more deeply about the task
Recording notes while monitoring

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Who and What</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number line</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comparing Decimal Equivalents</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Comparing Fractional Equivalents</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Selecting and sequencing

- **Selecting**
  - This is the process of deciding the *what* and the *who* to focus on during the discussion.

- **Sequencing**
  - What order will the solutions be shared with the class?
Connecting

- Perhaps the most challenging part
- Teacher must ask the questions that will make the mathematics explicit and understandable
- Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations
- Not just clarifying and probing
Teacher Questioning

Teachers can effectively use questions during the whole class discussion to help students to make deeper mathematical connections.
Moves to Guide Discussion and Ensure Accountability

- Revoicing
- Asking students to restate someone else’s reasoning
- Asking students to apply their own reasoning to someone else’s reasoning
- Prompting students for further participation
- Using wait time

Adapted from *5 Practices for Orchestrating Productive Mathematics Discussions*, Smith and Stein, NCTM, 2011
Purposeful Discourse

- Through mathematical discourse in the classroom, teachers “empower their students to engage in, understand and own the mathematics they study.”

(Eisenman, *Promoting Purposeful Discourse*, 2009)
Exploring Bivariate Data

Is there a relationship between the year and the winning time for the 200m men’s dash?
Olympic Gold-Medal Times for the 200-Meter Dash

<table>
<thead>
<tr>
<th>Year</th>
<th>Time in Seconds</th>
<th>Year</th>
<th>Time in Seconds</th>
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</thead>
<tbody>
<tr>
<td>1900</td>
<td>22.2</td>
<td>1956</td>
<td>20.6</td>
</tr>
<tr>
<td>1904</td>
<td>21.6</td>
<td>1960</td>
<td>20.5</td>
</tr>
<tr>
<td>1908</td>
<td>22.6</td>
<td>1964</td>
<td>20.3</td>
</tr>
<tr>
<td>1912</td>
<td>21.7</td>
<td>1968</td>
<td>19.8</td>
</tr>
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<td>1920</td>
<td>22.0</td>
<td>1972</td>
<td>20.0</td>
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<td>21.6</td>
<td>1976</td>
<td>20.23</td>
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<tr>
<td>1928</td>
<td>21.8</td>
<td>1980</td>
<td>20.19</td>
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<td>1932</td>
<td>21.2</td>
<td>1984</td>
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<td>1948</td>
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<td>1992</td>
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<tr>
<td>1952</td>
<td>20.7</td>
<td>1996</td>
<td>19.32</td>
</tr>
</tbody>
</table>
Exploring Olympic Times

- As a whole group ...
  - What do you notice about the data?
- On your own ...
  - Complete the part 1 worksheet
- In a small group ...
  - Share your solutions, strategies, and justifications
Is there a relationship between the year and the winning time? Explain.

Predict the winning times for 1916, 2000, 2012.

Compare the actual winning times for 2000 and 2012 with your predicted times.
Is the data linear?

- There are two ways to analyze how well the line fits the data—graphically and numerically.
  - **Graphically:** Draw the residual plot.
  - **Numerically:** Calculate the value of $r$, the correlation coefficient.
Is the Data Linear?

- **Graphically:** Draw the residual plot.
  - A residual = actual value – predicted value.
  - The residual plot will show the residual for each value of the independent variable.
Is the Data Linear?

- value of $r$, the correlation coefficient.
  - The closer this number is to 1 or $-1$, the more linear the relationship is. **Correlation coefficients** measure the strength of association between two variables.
  - In addition, $r^2$ is called the **coefficient of determination**. It gives the percent of the variation in the dependent variable that can be explained by the linear relationship.
Exploring Olympic Times

☐ As a whole group …
  ▪ What are residuals and correlation coefficients?

☐ On your own or in pairs …
  ▪ Complete the part 2 worksheet

☐ In a small group …
  ▪ Share your solutions, strategies, and justifications
Interpreting the Residuals and Correlation Coefficients

- How well did a linear model fit the time vs. year graph? Explain your reasoning.
Interpreting the Residuals and Correlation Coefficients
Interpreting the Residuals and Correlation Coefficients
Olympics Activity

- What question did we investigate?
- How did we collect data?
- How did we organize and analyze the data?
- What interpretations did we make about the data?
Discussion

☐ How does the process of statistical investigation, using the PCAI model, have the potential to support students’ in engaging in the SMPs?

☐ Which grade level standards have the potential to be addressed by the Olympics activity?

☐ How might you modify the activity for your specific students?
Our school’s basketball coach plans to give an award to the MVP. She is having a difficult time deciding which player is most worthy of the award. Based on the data, determine which of the three players should be given the MVP award.

Adapted from Mathscape’s (2002) Looking Behind the Numbers
Help the basketball coach determine which of the three players should be given the MVP award. Write a letter to the coach to convince her of your argument.

<table>
<thead>
<tr>
<th>Game</th>
<th>Player A</th>
<th>Player B</th>
<th>Player C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>18</td>
<td>24</td>
</tr>
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<td>13</td>
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<td>14</td>
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<td>15</td>
<td>18</td>
<td>11</td>
</tr>
</tbody>
</table>

Adapted from Mathscape’s (2002) *Looking Behind the Numbers*
Resources for Box Plots

NCTM’s Illuminations (Advanced Data Grapher)
http://illuminations.nctm.org/ActivityDetail.aspx?ID=220
Shodor (Box Plot)
http://www.shodor.org/interactivate/activities/BoxPlot/
What types of arguments do you expect students to make based on the data?

Watch the videos (#5, 9, 10).
- How did students engage in the SMP?
- Did anything surprise you?
- Based on our discussions in the last component on discourse, how might teachers support students in engaging in productive mathematical discourse?

http://mmmproject.org/lbn/mainframeS.htm
Questions/ Comments

Have a great school year!

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