

Binary Programming Formulation to Solve Sudoku Puzzle

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Variables

Let $A = [a_{i,j}]$ be the initial (incomplete) Sudoku puzzle in matrix form,

$A^0 = [a^0_{i,j}]$ be the solved Sudoku puzzle in matrix form,

$I = \{1, 2, \dots, K, 9\}$ be the row index set,

$J = \{1, 2, \dots, K, 9\}$ be the column index set,

$k \in K = \{1, 2, \dots, K, 9\}$ and $x_{i,j,k} \in \{0, 1\}$ be (implicitly) defined as follow

$$x_{i,j,k} = \begin{cases} 1, & \text{if } a^0_{i,j} = k \\ 0, & \text{otherwise} \end{cases}$$

Constraints

$$\sum_{i \in I} x_{i,j,k} = 1, \text{ for all } j \in J \text{ and } k \in K$$

$$\sum_{j \in J} x_{i,j,k} = 1, \text{ for all } i \in I \text{ and } k \in K$$

$$\sum_{k \in K} x_{i,j,k} = 1, \text{ for all } i \in I \text{ and } j \in J$$

$$\sum_{\substack{1 \leq i \leq 3 \\ 1 \leq j \leq 3}} x_{i,j,k} = 1, \text{ for all } k \in K,$$

$$\sum_{\substack{1 \leq i \leq 3 \\ 4 \leq j \leq 6}} x_{i,j,k} = 1, \text{ for all } k \in K,$$

$$\sum_{\substack{1 \leq i \leq 3 \\ 7 \leq j \leq 9}} x_{i,j,k} = 1, \text{ for all } k \in K$$

$$\sum_{\substack{4 \leq i \leq 6 \\ 1 \leq j \leq 3}} x_{i,j,k} = 1, \text{ for all } k \in K,$$

$$\sum_{\substack{4 \leq i \leq 6 \\ 4 \leq j \leq 6}} x_{i,j,k} = 1, \text{ for all } k \in K,$$

$$\sum_{\substack{4 \leq i \leq 6 \\ 7 \leq j \leq 9}} x_{i,j,k} = 1, \text{ for all } k \in K$$

$$\sum_{\substack{7 \leq i \leq 9 \\ 1 \leq j \leq 3}} x_{i,j,k} = 1, \text{ for all } k \in K,$$

$$\sum_{\substack{7 \leq i \leq 9 \\ 7 \leq j \leq 9}} x_{i,j,k} = 1, \text{ for all } k \in K$$

$$x_{i,j,k} = 1, \text{ if } a_{i,j} = k$$

$$\sum_{\substack{7 \leq i \leq 9 \\ 4 \leq j \leq 6}} x_{i,j,k} = 1, \text{ for all } k \in K,$$

Objective function

$$\max \sum_{i \in I, j \in J, k \in K} x_{i,j,k}$$

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