

Interrater Reliability in Healthcare Studies:

***Calculating the
Intraclass Correlation
Coefficient (ICC) in SAS***

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Interrater Reliability

- Each subject assessed by multiple raters
- To what extent are the ratings within a subject homogeneous?
- Ideally, want raters to be interchangeable

Decayed, Missing, Filled Teeth

Patient	Examiner			
	1	2	3	4
1	8	7	11	7
2	13	11	15	13
3	0	0	2	1
4	3	6	9	6
5	13	13	17	10
6	19	23	27	18

Intraclass Correlation Coefficient

- For continuous data, ICC often used to assess interrater reliability
- ICC is the correlation between two measurements made on same subject

$$ICC = \text{Corr}(Y_{ij}, Y_{ik})$$

ICC Properties

- Like any correlation, $-1 \leq \text{ICC} \leq 1$
- Ideally, ICC will be ≥ 0
- ICC Values close to 1 are desirable and indicate good interrater reliability

About the ICC

“There are numerous versions of the intraclass coefficient (ICC)...Each form is appropriate for specific situations defined by the experimental design...”

Shrout, PE and Fleiss, JL (1979) “Intraclass Correlations: Uses in Assessing Rater Reliability”, *Psychological Bulletin*, 86:2, 420-428

Randomized Block Design

- Dental example: 4 different raters randomly selected to rate each patient
- Patients represent random sample of all possible patients

$$Y_{ij} = \mu + \beta_i + \varepsilon_{ij}$$

$\beta_i \sim N(0, \sigma_\beta^2)$ random subject effect

$\varepsilon_{ij} \sim N(0, \sigma^2)$ experimental error

ICC for Randomized Blocks

$$\begin{aligned} Cov(Y_{ij}, Y_{ik}) &= E(Y_{ij}Y_{ik}) - E(Y_{ij})E(Y_{ik}) \\ &= \mu^2 + E(\beta_i^2) - \mu^2 \\ &= \sigma_\beta^2 \end{aligned}$$

$$Var(Y_{ij}) = Var(Y_{ik}) = \sigma_\beta^2 + \sigma^2$$

$$\therefore Corr(Y_{ij}, Y_{ik}) = \frac{\sigma_\beta^2}{\sigma_\beta^2 + \sigma^2} = ICC$$

Estimating the ICC: Method 1

● PROC GLM

1. Output mean squares to dataset
2. Use mean squares to calculate estimates of σ_{β}^2 and σ^2
3. Use the estimates of σ_{β}^2 and σ^2 to calculate the ICC

Estimating the ICC: More Methods

- **PROC MIXED**

1. Output estimates of variance components (part of standard output) to a dataset
2. Use the estimates to calculate ICC

- **PROC NLMIXED**

1. Calculate ICC within the procedure in a single step


- **%INTRACC macro**

1. No programming to do!

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Start with PROC MIXED + PROC SQL
Approach

```
ods output CovParms = cov1 ;  
proc mixed data=dental method=ml ;  
  class patient ;  
  model score = ;  
  random patient ;  
run ;
```



REML not available in
NLMIXED. For
comparison purposes, I
will use ML, and then
repeat with REML

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The Mixed Procedure

Covariance Parameter Estimates


Cov Parm	Estimate (ML)	Estimate (REML)
patient	41.8611	50.5528
Residual	6.3889	6.3889

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```
proc sql ;  
    create table icc as  
    select sum(estimate*(covparm='patient'))  
           / sum(estimate) as icc  
    from cov1 ;  
quit ;  
proc print data = icc ; run ;
```

Obs	icc (ML)	icc (REML)
1	0.86759	0.88780

Good reliability in
both cases




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PROC NLMIXED Approach

```
PROC NLMIXED data=dental method=firo ;  
  parms mu=10  s_subj = 50 s_err = 6 ;  
  pred = mu + beta ;  
  model score ~ normal(pred, s_err) ;  
  random beta ~ normal(0, s_subj)  
    subject = patient ;  
  estimate 'icc'  
    s_subj/(s_subj+s_err) ;  
  
run ;
```

Label	Estimate
icc	0.8676

Same estimate
as PROC
MIXED using
ML



Don't want to do the programming?

- There is a user-written macro that can be downloaded from the SAS website
- <http://support.sas.com/kb/25/031.html>
- The macro is called %INTRACC
- Computes 6 different versions of the ICC

%INTRACC and the Dental Data

- Macro invocation:

```
%intracc(depvar=score,  
         target=patient,  
         rater=rater, nrater=4)
```

- Documentation states that macro computes 6 different versions of ICC
- Output includes 9 versions

%INTRACC Output

Winer reliability: single score	Winer reliability: mean of k scores	Winer reliability: mean of 4 scores	Shrout-Fleiss reliability: single score	Shrout-Fleiss reliability: random set
0.88780	0.96937	0.96937	0.88780	0.88958
Shrout-Fleiss reliability: fixed set	Shrout-Fleiss reliability: mean k scores	Shrout-Fleiss rel: rand set mean k scrs	Shrout-Fleiss rel: fxd set mean k scrs	
0.94996	0.96937	0.96990	0.98700	

- Same ICC value as PROC MIXED using REML

Dental Data Using PROC GLM

- First, output the mean squares to a dataset
- Next, examine expected mean squares to determine relationship between MS's and variance components, σ_{β}^2 and σ^2

GLM Code

```
ods output ModelANOVA=ms1
      (where=(hypothesistype=3)) ;
ods output OverallANOVA=ms0
      (where=(source='Error')) ;
proc glm data = dental ;
      class patient ;
      model score = patient ;
      random patient ;

run ;
```

GLM Continued

The GLM Procedure

Source	Type III Expected Mean Square
patient	$\text{Var}(\text{Error}) + 4 \text{Var}(\text{patient})$

σ^2

σ_{β}^2

- So, use MS(error) to estimate σ^2
- Use $[\text{MS}(\text{patients}) - \text{MS}(\text{error})]/4$ to estimate σ_{β}^2

GLM Continued Again

```
proc sql ;  
  create table icc as  
  select a.ms as s_err, (b.ms-a.ms)/4 as s_sub,  
         ((b.ms-a.ms)/4) / (a.ms+(b.ms-a.ms)/4)  
         as icc  
  from ms0 a, ms1 b;  
quit ;  
proc print data = icc ; run ;
```

Obs	s_err	s_subj	icc
1	6.388889	50.5528	0.88780

Same estimates as
PROC MIXED with
REML



Comparison for RCB Design

- GLM is cumbersome!
- %INTRACC is too much of a black box for me
- NLMIXED is simple, which I like, but no option to do REML
- MIXED seems to me like best compromise
- What about more complicated designs?

Random Rater Effect

- Suppose same 4 raters assess each subject, but that the 4 raters randomly selected from larger pop'n of raters

$$Y_{ij} = \mu + \beta_i + \tau_j + \varepsilon_{ij}$$

$\beta_i \sim N(0, \sigma_\beta^2)$ random subject effect

$\tau_j \sim N(0, \sigma_\tau^2)$ random rater effect

$\varepsilon_{ij} \sim N(0, \sigma^2)$ experimental error

ICC for Random Rater Effect

$$\begin{aligned} Cov(Y_{ij}, Y_{ik}) &= E(Y_{ij}Y_{ik}) - E(Y_{ij})E(Y_{ik}) \\ &= \mu^2 + E(\beta_i^2) - \mu^2 \\ &= \sigma_\beta^2 \end{aligned}$$

$$Var(Y_{ij}) = Var(Y_{ik}) = \sigma_\beta^2 + \sigma_\tau^2 + \sigma^2$$

$$\therefore Corr(Y_{ij}, Y_{ik}) = \frac{\sigma_\beta^2}{\sigma_\beta^2 + \sigma_\tau^2 + \sigma^2} = ICC$$

NLMIXED and GLM

- Oops!! NLMIXED can't handle “crossed” random effects. Maybe in an upcoming version?
- Here are the expected mean squares from PROC GLM:

The GLM Procedure

Source	Type III Expected Mean Square
patient	$\text{Var}(\text{Error}) + 4 \text{ Var}(\text{patient})$
rater	$\text{Var}(\text{Error}) + 6 \text{ Var}(\text{rater})$

I'm too lazy to do the calculations so
I'll skip ahead to PROC MIXED

MIXED For Random Rater Effect

```
ods output CovParms = cov2 ;  
proc mixed data = dental method = REML ;  
  class patient rater ;  
  model score = ;  
  random patient rater ;  
run ;  
proc sql ;  
  create table icc2 as  
  select sum(estimate*(covparm='patient'))  
  / sum(estimate) as icc  
  from cov2 ; quit ;
```

Obs	icc
1	0.88958

%INTRACC Output

Winer	Winer	Winer	Shrout-Fleiss	Shrout-Fleiss
reliability:	reliability:	reliability:	reliability:	reliability:
mean of	mean of	mean of	single score	random set
single score	k scores	4 scores		
0.88780	0.96937	0.96937	0.88780	0.88958
Shrout-Fleiss	Shrout-Fleiss	Shrout-Fleiss	Shrout-Fleiss	
reliability:	reliability:	rel: rand set	rel: fxd set	
fixed set	mean k scores	mean k scrs	mean k scrs	
0.94996	0.96937	0.96990	0.98700	

- Same macro call as earlier & same output
- Same value as MIXED with REML

Raters from 2 Different Specialties

- Suppose that 1st two raters are from one dental specialty & 2nd two raters are from another
- Only two specialities of interest
- \therefore speciality is a fixed effect
- 2 raters picked at random from each specialty
- Rater effect expected to differ by specialty

Model for Different Specialties

$$Y_{ijk} = \mu + \beta_i + \tau_{j(k)} + \gamma_k + \varepsilon_{ij(k)}$$

$\beta_i \sim N(0, \sigma_\beta^2)$ random subject effect

$\tau_{j(1)} \sim N(0, \sigma_{\tau_1}^2)$ random rater effect for 1st specialty

$\tau_{j(2)} \sim N(0, \sigma_{\tau_2}^2)$ random rater effect for 2nd specialty

γ_k fixed specialty effect

$\varepsilon_{ij} \sim N(0, \sigma^2)$ experimental error

ICC for this Model

Same subject, same specialty:

$$ICC = \frac{\sigma_{\beta}^2}{\sigma_{\beta}^2 + \sigma_{\tau k}^2 + \sigma^2}$$

Same subject, different specialties:

$$ICC = \frac{\sigma_{\beta}^2}{\sqrt{\sigma_{\beta}^2 + \sigma_{\tau 1}^2 + \sigma^2} \times \sqrt{\sigma_{\beta}^2 + \sigma_{\tau 2}^2 + \sigma^2}}$$

Challenge

- Expressions for Model and ICC are easy to write out, but how to fit the model and estimate ICC?
- “group=specialty” option in MIXED varies *all* parameters by specialty instead of just the rater variance
- NLMIXED would allow this if there was only one random effect, but there are 2, which it can't handle